# Buyout Options in Simultaneous Multi Unit Auctions

**Dissertation**
**for the Faculty of Economics, Business Administration and Information Technology of the University of Zurich**

to achieve the title of
Doctor in Economics

presented by

Marcel Sweys
from Meiringen (BE)

approved at the request of

Prof. Dr. Dieter Pfaff
Prof. Dr. Conrad Meyer

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The Faculty of Economics, Business Administration and Information Technology of the University of Zurich herewith permits the publication of the aforementioned dissertation without expressing any opinion on its views.

Zurich, December 5, 2007

The Dean: Prof. Dr. H. P. Wehrli

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# Chapter 1

# Introduction

In late 2000, eBay introduced a feature called "Buy it Now", marking a cornerstone in the way goods are being auctioned by allowing for a unique hybris of both traditional auction and fixed price mechanisms. The institution offered on eBay is in fact a Buyout Option that allows any bidder to terminate an auction without the necessity of traditionally bidding in an auction. Buyout Options in auctions have since its early adoption on eBay experienced phenomenal popularity on numerous online auction markets and have to date been implemented on most such platforms while they are still being rarely observed in classic auctions.

The implementation of optimal auction mechanisms has been a key issue in the long lasting scientific and political debate amongst economists and has put forth a tremendous quantity of theoretical, empirical and experimental research, long before goods have been auctioned on electronic market places. The emergence of momentous and easily accessible auction markets however makes an advancement and progression of auction mechanisms indispensable for its current meaningful position in commerce not to irretrievably lose ground. Moreover, the rapid growth of online auction markets seen in preceding years has not been estimated to slow down in the near future. While the diffusion and use of the internet remains the driving force for the growth of electronic based auction markets, further development of auction mechanisms is a prerequisite. Furthermore, with regard to financial markets, auctions are perceived to gain momentum on its primary markets due to their preeminent economic advantages versus other sales practices, not to mention the use of auction mechanisms in numerous other areas of commerce.

Buyout Options in auctions however have only recently attracted the interest of economists. The existence and increasing popularity of such options in auctions seem peculiar from the point of view of auction theory since they may detriment the auction's primary benefit of relieving the seller to determine the price for the good and allowing for competition amongst bidders. The introduction of a Buyout Option may further reduce seller's expected revenue because when it is being exercised, it may eliminate the possibility of higher prices that would have been reached by competitive bidding. Buyout Options may as well lead to allocative inefficiencies when they are being exercised by bidders who do not value the goods at sale most.

While such options are increasingly being used in practice, literature for the time being still lacks much of the analysis of Buyout Options in auctions, let alone its examination for the case of simultaneous multi unit auctions that has so far not been addressed by auction theorists. The theoretical literature dealing with Buyout Options in auctions has to date predominantly focused on the analysis of single unit auctions enhanced by such an option, notwithstanding the fact that they can be used and have indeed already been implemented for simultaneous multi unit auctions in practice.

Motivated by the recent surge of literature on such options in auctions, the aim of this thesis is to fill this gap and contribute to the literature by introducing a model of a simultaneous multi unit auction enhanced with a Buyout Option. Its research question is whether it does indeed make sense to allow a seller to offer such an option when simultaneously offering identical goods for sale and what its implications are on both the seller's expected utility and allocative efficiency. Actually, if such options were found to be beneficial for either side of the transaction (or both), auctioneers should avail themselves of the opportunity to allow for the implementation of Buyout Options in simultaneous multi unit auctions. Due to the complexity of Buyout Options in auctions it is however beyond the scope of this thesis to draw a comprehensive conclusion of all possible implementations of Buyout Options in multi unit auctions. It is rather the purpose of this study to illustrate that even for the case where several homogeneous goods are simultaneously being offered in an auction, it may effectively make sense to offer such an option.

If it can be shown that despite the potential allocative inefficiencies and loss of seller revenue arising with such options in place, an enhancement of a simultaneous multi unit auction by a Buyout Option is indeed beneficial for sellers and bidders alike, these findings would not only provide support for the actual use of such options but would further have important implications for the understanding of its effects on the outcome of auctions where the seller simultaneously tenders more than just a single good.

The remainder of this thesis is organized as follows. Chapter 2 provides an overview on fundamental auction theory and its key findings. Chapter 3 then introduces the taxonomy and characteristics of Buyout Options in auctions and illustrates its potential benefits and drawbacks for both the seller and the bidders. Furthermore, a brief survey on the use of Buyout Options in a selection of real world auctions is given to exhibit its practical significance. The aim of chapter 4 is to provide a survey on existing theoretical, empirical and experimental literature on Buyout Options in auctions. Chapter 5, which, along with chapter 6, is the core of this study, presents an analytical model of a simultaneous multi unit auction enhanced with a Buyout Option. A model is being formulated that allows for the analysis of a simultaneous multi unit auction enhanced with a temporary Buyout Option. Optimal equilibrium strategies for a risk averse seller and risk neutral bidders are derived, using an independent private values framework. Chapter 6 then discusses the results and findings from the model. The main contributions as well as limitations of this work are summarized in chapter 7, concluding with a discussion on future research opportunities in the field.

# Chapter 2

# Auction Theory

In this chapter, a brief overview of auction theory is given to facilitate the classification and comprehension of common auction mechanisms as market institutions with explicit rules that determine specific prices and the allocation of goods based on bids submitted, as well as the model subsequently discussed. A focus is then being put on specific topics that can be observed in real world auctions and have fostered recent advances in auction theory. It should however be noted that the presentation given herein is by no means exhaustive since a comprehensive formal discussion of all topics in auction theory is beyond the scope of this thesis. Readers are advised to refer to the sources indicated in the text for further reference on the topics addressed hereafter.[1]

Auctions have increasingly gained popularity on various markets due to their preeminent merits. The dynamic and flexible pricing features of auctions not only allow sellers to potentially gain higher revenues than on fixed price markets but also allow for the achievement of economic efficiency in that the bidder who most values the good at sale receives the good, a criterion oftentimes not met by fixed price mechanisms. Furthermore, due to asymmetric information on the market value of numerous goods amongst market participants, auctions are a valuable tool in view of its price discovery power.

The fundamental even though not exclusive questions addressed in auction theory are based on the ground of two primary yet distinctive measures, the pertinence of each of them depending on the specific context. The first aspect is from the seller's perspective with regard to the *revenue* that can be raised in different auction

[1] For a comprehensive overview on auction theory, see Engelbrecht-Wiggans (1980), Reiley and Samuelson (1981), Maskin and Riley (1984), McAfee and McMillan (1987), Milgrom (1987b), Wilson (1992), Wolfstetter (1996), Klemperer (1999), Krishna (2002), Klemperer (2004), Menezes and Monteiro (2005) and Milgrom (2006).

formats. Apparently, the seller's primary objective is to adopt a mechanism that leads to the highest expected sales price. Second, from the perspective of the bidders or the market as a holistic institution, the *efficiency* of different mechanisms, that is, that the goods at sale are awarded to the individuals who value them most, is key. In fact, as will be shown in what follows, different auction mechanisms can have highly characteristic features that depend on the context in which they are applied, and potentially lead to very distinctive outcomes.

## 2.1 Brief History of Auctions

Babylonian wedding auctions as early as 500 B.C. were among the first auctions documented.[2] In these annually held auctions, interested bridegrooms could bid for women for a future marriage by way of a descending price auction. The auctioneers started at a high price that they continually lowered until a bidder accepted the maiden. It has been recorded that the prices paid for the women often related to their beauty and the less comely women not seldom had to pay a substantial dowry to be accepted, thereby making the actual price negative in many cases. In the Roman Empire, auction mechanisms were also widely used for the sale of the "spoils of the war" to the soldiers after successful warfares. Compared to the auctions held by the Babylonians, the Roman auctions were much more sophisticated and organized with regulated hosts and promoters who advertised and auctioned the goods. In the Roman Empire, auctions were also extensively used to liquidate personal properties. Marcus Aurelius is told to have sold precious furniture and heirlooms in auctions that lasted up to two months. The etymological source of the word "auction" can also be traced back to the Romans since it is derived from "auctus", being the past participle of "augere", which means "to increase".

King Henry VII. of England later in the Middle Ages of the fifteenth century instituted some of the earliest forensic auction regulations such as including auction licenses that indeed legally permitted vendors to sell their goods by way of an auction. In the early seventeenth century, auctions became much more common in Great Britain to sell pieces of art and other collectibles. Only a short while later, the two auction houses Sotheby's and Christie's were found in 1744 and 1766 respectively which are dazzling names to this very day. After the colonialization of America, auctions were widely used for the sale of grains, tobacco, land and slaves. Early accounts of the use of auctions in the Netherlands and Germany, where fruits, veg-

[2] See Cassady (1967) for a comprehensive overview of the history of auctions.

etables and flowers were sold, endorse its adoption in the later part of the nineteenth century.

One of the most prominent contemporary institutions using an auction mechanism instead of a fixed price market is the U.S. Treasury Department to issue debt on the primary market. The liquidity and volume of U.S. public debt securities that are continuously and extensively being traded on global financial markets points to the importance of its sale on the primary market and not least to the relevance of auctions on financial markets.[3] Another increasingly significant use of auctions on financial markets is with regard to the issue of shares on primary markets.[4] Google's initial public offering in August 2004 marked a breakthrough of auction mechanisms used on the issue market for equities despite the fact that only few issuers have so far decided to follow its path. Future however will show whether auctions will gain importance on financial markets as they offer the best conceivable means for both sides of such transactions.

The growth of the number of internet auction sites as well as its trading volume is ultimately the paradigm of today's importance of auctions in everyday life. To exemplify the relevance of online auction markets, the volume of goods sold on eBay has experienced a remarkable growth over the past few years, starting from a volume of only USD 1.15 billion in gross merchandise sales in the first quarter of 2000 to an amount of USD 14.5 billion in gross merchandise sales in the second quarter of 2007.[5] The substantial evolution of the overall nominal value of goods sold on eBay is apparent from Figure 2.1.

Online auction markets offer various remarkable advantages that gave rise to their present-day status and continuous growth. Amongst others, such markets substantially reduced transaction costs associated with operating and participating in an auction.[6] Moreover, the comfortable accessibility to such auctions along with the remarkable diffusion of the internet over the past few years has sharply increased the spectrum of potential sellers and buyers that can meet virtually instead of the necessity of physically being present and thereby raised the probability of bidders with high valuations to effectively participate.[7] Another noteworthy fact that eases

[3] See Berney (1964), Smith (1966), Back and Zender (1993), Ausubel and Cramton (1998) and Ausubel (2002).
[4] See Kandel et al. (1999), Biais and Faugeron-Crouzet (2002) and Sherman (2005).
[5] See eBay's quarterly financial results.
[6] See Ockenfels et al. (2006).
[7] Another form of "distant" bidding has however already gained importance in traditional auctions before electronic markets emerged by allowing bidders to participate by telephone.

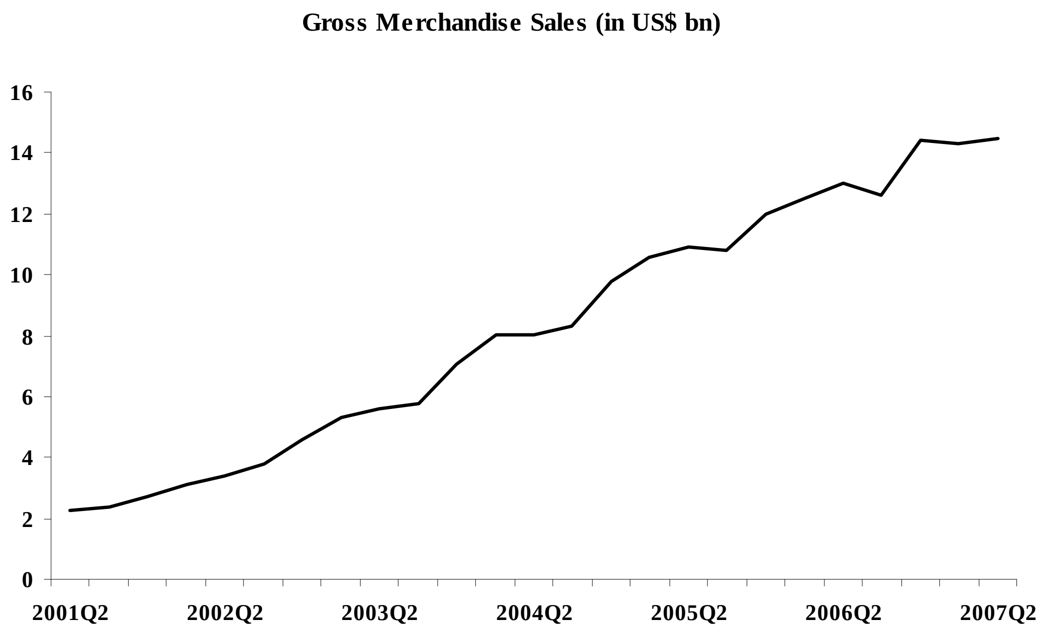


Figure 2.1: Gross Merchandise Sales on eBay, 2001Q2 - 2007Q2

the participation in online auctions is that most platforms offer electronic bidding agents that enable bidders to submit their preferences before an auction takes place (or during an auction) and therefore even eliminate the need to attend the auctions at the scheduled time.[8] The growth rates and increasing importance of such online auction platforms for commerce has ultimately boosted momentum in auction theory, making it one of the most buoyant research areas in economics.

## 2.2 Auction Types

Four basic types of auctions are widely used and analyzed in existing theoretical literature. In describing the mechanisms of these auction types, for simplicity, the focus is first being put on auctions where a single indivisible good is at sale before turning to an overview of auctions where multiple units are being sold. The four fundamental auction types are: (i) ascending price auctions, (ii) descending price

[8] The ability to submit bids by an agent or automatic bidding mechanism is called "proxy bidding" in auction literature. Bidders can submit the maximum amount they are willing to pay for the good to a bidding agent. This information is however not disclosed to any of the other auction participants. The agent will then place bids on the bidder's behalf, submitting only an amount at the lowest possible increment that is necessary to outbid all other bidders, given that the maximum bid submitted to the agent exceeds all other current bids. If another bidder submits a bid higher than the maximum amount entered, such a bidder will be outbid. When no other bidder submits a higher bid until the end of the auction, the bidder is deemed successful and receives the good.

auctions, (iii) first price sealed bid auctions and (iv) second price sealed bid auctions. As the categorization of these four fundamental auction types reveals, the basic taxonomy of their denomination is based upon the order in which prices are quoted and in the way individual bids are tendered.

The most widely used auction type is the *ascending price auction*, also known as English auction or open outcry auction, where the price is increased by the seller from a relatively low level where multiple bidders are willing to acquire the good until only one bidder remains. The bidder who last remains in the auction wins the good at the price where his latest rival dropped out. If the seller posts a reserve price, only bids at or above this ex ante specified minimum required price level are being accepted and if no bidder posts a bid at or above the reserve price, the good is not sold. The simplicity of ascending price auctions thus makes them very attractive for practical use.

The *descending price auction* or Dutch auction works in the opposite way of an ascending price auction in that the seller starts at a relatively high price level, at which presumably no bidder will buy the good, that is then subsequently being lowered. The bidder who first accepts a price wins the good and pays the price at which the auction stopped. Therefore, the descending price auction is often referred to as the converse of the ascending price auction, despite its evident significant distinctions. When comparing with the ascending price auction where the price is specified by the second strongest bidder, the final price in the descending price auction is solely determined by the bidder who first accepts to buy the good and thus by the winning bidder. Note that the term "Dutch Auction" has recently widely been used in finance literature and on multiple online auction sites to refer to a type of auction other than the descending price auction, that is, the term is extensively being used to refer to what is defined as a multi unit uniform price auction in auction literature, thereby potentially leading to great confusion.

In contrast to the two first auction types, in a *first price sealed bid auction* individual bidders do not receive information about the other bids during the auction since they are all being independently submitted to the seller by sealed bids. The bidder with the highest bid wins the auction and pays a price equal to his bid.

Finally, in a *second price sealed bid auction* or Vickrey auction, bidders also independently submit their bid as in the first price sealed bid auction. However, here the bidder with the highest valuation wins the good and only pays a price equal to the second highest bid submitted. Obviously, in this type of an auction, the final price is determined by the second highest bid.

## 2.3 Auction Models

A key feature of commerce in general and especially in auction practice is the presence of asymmetric information. Bidders competing in auctions may have differing valuations for the good at sale that are ex ante not entirely revealed to the seller or to the other competing bidders. Likewise, the seller's valuation for the good may be withheld and not revealed to the bidders. If there was full disclosure of all participants' valuations for the good, the seller could anticipate the outcome of the auction and behave appropriately to fully extract the bidders' rent and thereby maximizing his revenue. The seller could then simply make a fixed price offer to the bidder with the highest valuation at or just below his valuation. However, in most transactions the buying parties' valuations for the good are not common knowledge. Auctions are a formidable mechanism for a seller who is uncertain about the values that each bidder attaches to the good being sold to extract the highest possible revenue. Auctions can hence be classified into three distinct models. These are: (i) independent private values auctions, (ii) common values auctions and (iii) interdependent values auctions.

In *independent private values auctions* the individual bidders have independent and private valuations for the good at sale. Thus, different bidders may have different valuations for the good that are not preliminarily known with certainty by the seller or by the other bidders. Recall that if the seller knew the individual bidders' valuations, he could simply make a take-it-or-leave-it offer to extract the highest possible rent from the bidder with the highest valuation. Furthermore, there is asymmetric information amongst the bidders since they do not know the valuations of their respective competitors, leading to possibly material consequences for their optimal bidding behaviour. Moreover, in the independent private values model, bidders' valuations are unaffected by signals or additional information on the value of the good from any other bidder that may be revealed during the auction process. Thus, even if the individual valuations were common knowledge among all auction participants, each individual bidder's valuation would be unaffected. Independent private values auctions are most plausible for goods whose values are derived from its consumption or possession alone and not on the basis of a possible resale market.

As opposed to independent private values auctions, in the *common values auction* model, the good to be sold has the same ex ante value to every bidder. Nevertheless, individual bidders have different information about the actual or "true" value of the good and therefore its value is not known with certainty at the time of the commencement of the auction. Thus, since the value of the good is not known by the bidders before the conclusion of the auction, an individual bidder may gain information about the true value of the good by observing other bidders' behaviour during the auction or by receiving additional information on the good. Such a bidder can then accordingly update his estimate of the good's true value, and the auction to a certain extent follows the principle "felix qui potuit rerum cognoscere causas".[9] Most auctions in which bidders intend to resell the good after successful participation at any given subsequent stage after the conclusion of the auction on a secondary market are common values auctions.

The *interdependent values auction* model is a more general model that includes both the private and common values models as special cases. The interdependent values model is an integrative approach that considers auctions in which bidder valuations depend on their individual preferences, the preferences of other bidders and the specific characteristics of the good to be sold. A bidder's valuation of the good does thus depend on the information received on other bidders' valuations and from the good itself. In this setting, a high valuation of one bidder makes it more likely that other bidders have relatively high valuations for the same good if they receive information about the valuation of the bidder who values the good relatively high. Thus, interdependent value auctions are particularly suited for goods that can possibly be resold on any form of a market subsequent to the auction. When bidders fully ignore the information revealed during the auction, then the interdependent values model is equivalent to the independent private values model. On the other hand, if only a single measure on the quality of the good at sale is equally known by all bidders but no additional information on the good or the respective other bidders can be revealed, then the interdependent values model and the common values model are equivalent since all bidders identically base their valuations on that single measure.

In what follows, the different auction types and models are being presented for auctions where only a single good is at sale. Subsequently, auctions in which multiple homogeneous goods are tendered are being introduced. The main findings of auction theory can thereby easily be derived and illustrated in a comprehensive manner.

[9] Quoted from Vergil's Georgics, Book II. This phrase points to the fact that the bidder who was able to estimate the true value of the good best turns out to optimally submit a bid. A rough translation of the phrase could be: "Happy is the one who was able to understand the causes of things".

## 2.4 Single Unit Auctions

To commence, auctions in which only a single good is sold are being considered. Single unit auctions have extensively been analyzed and discussed in theoretical as well as empirical literature and have bred the findings perceived as amongst the most remarkable in auction theory. These fundamental results will now be described by applying an accessible and non-technical approach.

### Independent Private Values

To start with, note that given independently and identically distributed bidder valuations, the descending price auction and the first price sealed bid auction are strategically equivalent.[10] Despite the fact that the descending price auction is an open auction, the only information being revealed in the course of the auction is when a bidder accepts a price, thus not until the end of the auction. Therefore, a specific bid in the sealed bid first price auction is equivalent to a bid posted in a descending price auction since only the bidder submitting the highest bid wins the good in either case, paying the bid he previously submitted. Similarly, the ascending price auction is equivalent to the second price sealed bid auction since in both auction formats it is optimal for every bidder to bid up to his valuation, despite the fact that in an ascending price auction the information that some bidders drop out might reveal some information about the value of the good. This information is however useless if individual valuations are independent.[11]

Since given independent and private bidder valuations, the sealed bid first price auction is equivalent to the descending price auction and the ascending price auction is equivalent to the second price sealed bid auction, it suffices to solely consider one of the respective auction types when comparing the four auctions. Clearly, in a second price sealed bid auction it is a dominant strategy for every bidder to truthfully bid his valuation.[12] In a first price sealed bid auction however, no bidder would bid an amount equal to his valuation since then he could not gain any positive payoff as the maximum payoff he could in that case receive is zero. When participating in a first price sealed bid auction, a bidder needs to balance between increasing his bid up to his valuation and thereby increasing the probability of winning the

[10] Here and in what follows, it is assumed that bidders are risk neutral and symmetric, that is, the distribution of values is the same for all bidders. Further, auctions are considered to be strategically equivalent if for any bidder's strategy in one auction, there is a corresponding strategy in the other auction resulting in the same outcome. See Krishna (2002), p. 4.

[11] See Krishna (2002), pp. 15-16.

[12] See Vickrey (1961).

auction while at the same time reducing his expected payoff. Bidders will therefore tend to "shade" their bids and optimally bid an amount equal to or just above the second highest bidder's valuation and thus potentially lowering expected seller revenue. However, given that bidder valuations are independently and identically distributed and are private information, expected seller revenue is the same both in a second price sealed bid auction and in a first price sealed bid auction, that is, the expectation of the second highest bidder's valuation. Therefore, with independent and identically distributed private bidder valuations, expected seller revenue is the same in all four auction types. This result is known as the *Revenue Equivalence Theorem*, one of the most remarkable findings in auction theory.[13]

**Interdependent Values**

When relaxing the assumption of independent private bidder valuations and allowing for the possibility that bidders instead have partial information with regard to the value of the good and other bidders' valuations, the results obtained from the case of independent private valuations may substantially differ. Since with interdependent valuations, bidders do not know the actual value of the good with certainty at the beginning of the auction and may receive valuable information about the other bidders' valuations during the auction, they may need to revise their estimation of the value of the good which makes bidding strategies considerably more complex. With interdependent values, the winning of an auction may lead to a decrease of the estimated value of the good since a successful bidder eventually pays too much, a result referred to as the *Winner's Curse* in auction theory.[14] To avoid such an unfavourable outcome, bidders will therefore shade their bids well below their initial estimates for the good with positive probability which in turn might detriment seller revenue. Furthermore, ascending price auctions and second price sealed bid auctions are no longer strategically equivalent since bidders in an ascending price auction can indeed update their estimates on the true value of the good.[15] Also,

[13] See Vickrey (1961), Vickrey (1962), Myerson (1981) and Reiley and Samuelson (1981). The key assumptions underlying the Revenue Equivalence Theorem are (i) *independence* (the values of different bidders are independently distributed), (ii) *risk neutrality* (all bidders seek to maximize their expected profits), (iii) *no budget constraints* (all bidders have the ability to pay up to their respective values) and (iv) *symmetry* (the values of all bidders are distributed according to some specific distribution function). A violation of one of these assumptions will indeed affect the Revenue Equivalence Theorem and potentially invalidate it. See Krishna (2002), p. 37.

[14] See Ortega Reichert (1968), Wilson (1969), Capen et al. (1971), Wilson (1977), Kagel and Levin (1986), Thaler (1988), Thiel (1988), Harstad and Rotkopf (1995), Bulow and Klemperer (2002) and Bajari and Hortascu (2003).

[15] If however only two bidders participate in the auction, the ascending price auction is still equivalent to the second price sealed bid auction since when either of the bidder drops out, the auction is

the Revenue Equivalence Theorem will no longer hold with interdependent values due to the correlation of bidder valuations. In particular, the second price sealed bid auction will generate at least as much expected seller revenue as the first price sealed bid auction. Similarly, expected seller revenue in ascending price auctions weakly dominates expected seller revenue in second price sealed bid auctions. In auction theory, these findings are captured as the *Revenue Ranking Principle*.[16]

## 2.5 Multi Unit Auctions

When instead of only a single good several identical goods are auctioned, a seller again has various options when choosing an auction mechanism. This section aims to give a brief yet partial overview on fundamental principles and results from auction theory pertaining to multi unit auctions.[17]

Start by noting that when several homogeneous goods are being offered and bidders have multi unit demand, their bidding behaviour does critically depend on their marginal valuations of the goods, that is, they can either have decreasing marginal valuations (as for substitutes), constant marginal valuations or even increasing marginal valuations (as for complements).[18] Furthermore, the seller has different ways to sell the items, either one at a time in separate multiple auctions or simultaneously in a single auction.[19]

### Simultaneous Auctions

When multiple identical goods are simultaneously being offered in a single auction, the seller can choose between several distinct auction formats, amongst others three sealed bid auction formats. These are: (i) Multi unit discriminatory auctions, (ii)

over and the remaining bidder can no longer update his estimate of the true value. See Krishna (2002), p. 86.

[16] See Ortega Reichert (1968), Wilson (1969) and Milgrom and Weber (1982).

[17] For simplicity, only the case for independent private valuations and ex ante symmetric and risk neutral bidders is here being discussed. For further reference with regard to multi unit auctions with interdependent valuations, see Maskin (1992), Dasgupta and Maskin (2000), Jehiel and Moldovanu (2001) and Maskin (2001). Moreover, the discussion is in terms of discrete multi unit auctions. When alternatively extending the models to the continuous case, as for share auctions of a perfectly divisible good, the essential properties of the different models presented hereafter are not affected. See Wilson (1979), Back and Zender (1993) and Wang and Zender (1993).

[18] In the following, it is assumed that bidders' marginal valuations are declining, that is, the value of an additional good decreases with the number of goods already acquired. Therefore, bidders' demand is non-increasing in price. This situation is in extenso discussed in auction theory and can be applied to most real world multi unit auctions.

[19] The terms "good", "item" and "unit" are subsequently synonymously being used.

multi unit uniform price auctions and (iii) multi unit Vickrey auctions. These three auction formats, while all allocating the goods to the highest bids submitted, substantially differ in their pricing rules.

In a *multi unit discriminatory auction*, the highest submitted bids are deemed successful and every winning bidder pays an amount equal to the sum of his bids for which he was awarded a good. Therefore, this price determination mechanism amounts to perfect price discrimination since it can alternatively be seen as the extension of the first price sealed bid auction to multiple units (each bidder pays the price submitted by his demand function).[20] Obviously there is bid shading in this auction format since when instead bidding truthfully, there would be no gains from winning any good.[21] With positive probability, bidders even submit flat demand functions, that is, they bid the same price for each good. Most likely however, bidders submit downward sloping demand functions, that is, bidding lower prices for additional goods. As a consequence, equilibria in multi unit discriminatory auctions generally are inefficient.[22] However, with single unit demand, the multi unit discriminatory auction turns out to be efficient.

An example will help understand the price determination mechanism of a multi unit discriminatory auction. Consider the case where a seller offers five homogeneous goods for sale to three bidders that submit bid vectors $b_1 = (23, 17, 13, 11, 9)$, $b_2 = (16, 14, 12, 10, 8)$ and $b_3 = (15, 7, 5, 3, 2)$, where the subscript denotes the individual bidder and bid vectors are sorted in decreasing order where the highest value is the respective bid for the first unit, the second bid for the second unit, and so forth.[23] Thus, the five highest bids are $(23, 17, 16, 15, 14)$. Since in a multi unit discriminatory auction the respective highest bids are awarded with a good, bidder 1 receives two units and pays 23 for the first unit and 17 for the second unit. Bidder 2 receives two units as well and pays 16 and 14 for the first and second unit respectively. Bidder 3 only receives a single item at a price of 15. Thus, individual payoff is zero for all bidders in this example since they pay prices equal to their individual valuations for all goods obtained. Overall seller revenue is $23 + 17 + 16 + 15 + 14 = 85$.

[20] Further, note that if only a single good was auctioned, this auction format would naturally reduce to a single unit first price sealed bid auction.
[21] See Ausubel and Cramton (1996) and Engelbrecht-Wiggans and Kahn (1998b).
[22] See Vickrey (1961).
[23] In this example, it is assumed that the bids submitted for the individual goods are equal to the bidder's true valuations for the goods. Thus, bidders are assumed to be "naive" in that they do not strategically shade their bids. Further, assume that an individual bidder's payoff is the difference between his individual valuation and the price paid for the goods.

In contrast, in *multi unit uniform price auctions* or non-discriminatory multi unit auctions, the goods are sold at a unique price, often referred to as the "market clearing price", where aggregate demand equals supply.[24] In this auction format, the number of goods each bidder obtains equals the number of his competitors' bids he defeats.[25] In multi unit uniform price auctions, it is a dominant strategy for bidders to bid truthfully for the first good and shade bids for all additional goods since with positive probability every bid submitted by successful bidders other than the one for the first unit determines the final price effective for all units.[26] Thus, a successful bidder may himself influence the price he will eventually need to pay. Therefore, the fact that bidders shade their bids generally leads to an inefficient allocation.[27] However, when all bidders again only have single unit demand, they have no incentive to shade their bids since no winning bidder does influence the final price.[28] Thus, with single unit demand, even the multi unit uniform price auction is efficient.

Note that the general inefficiency in both the multi unit discriminatory auction and the multi unit uniform price auction does not arise from the fact that multiple units are being tendered but stems from the fact that bidders feature multi unit demand. When bidders instead have single unit demand, these two auction formats are even revenue equivalent.[29]

Considering again the example with the bidding vectors given above, in the case of the multi unit uniform price auction, the goods are identically awarded to the respective bidders with the highest bids submitted, but at a unique price of 13 (since this is the highest losing bid, submitted by bidder 1 for his third good requested). Thus bidder 1 and bidder 2 would each pay 26 and receive two goods while bidder 3 would receive a single good at the same price. Bidder 1's payoff is $23 + 17 - 2 \times 13 = 14$, bidder 2's payoff $16 + 14 - 2 \times 13 = 4$ and bidder 3's payoff amounts to $15 - 13 = 2$. Finally, seller revenue would amount to $5 \times 13 = 65$.

[24] Assume that the final price is equal to the highest losing bid.
[25] Further, note that when only a single good is being sold, the multi unit uniform price auction reduces to a single unit second price sealed bid auction.
[26] In extreme cases, the equilibrium price can even tend to zero. However, when the number of bidders exceeds the number of goods at sale, such low revenue equilibria cannot arise. See Noussair (1995) and Engelbrecht-Wiggans and Kahn (1998a).
[27] See Vickrey (1961).
[28] Note that when the final price is instead determined by the lowest winning bid, other outcomes are possible since then successful bidders can potentially influence the price.
[29] In this special case, the Revenue Equivalence Theorem can be applied.

In *multi unit Vickrey auctions*, a successful bidder pays the respective highest losing bids of all other bidders for every good awarded.[30] Therefore, similar to the multi unit discriminatory auction format, winning bidders do not pay a uniform price but here, prices are based on the other bidders' behaviour.[31] As it is the case for single unit Vickrey auctions, it is a dominant strategy to truthfully bid in multi unit Vickrey auctions and the auction allocates the objects efficiently.[32]

Given the bid vectors in the previous example, bidder 1 would again receive two goods but pay an amount equal to the other bidder's highest losing bids, that is, he would pay 12 for the first good and 10 for the second good and receive a payoff of $23 + 17 - 12 - 10 = 18$. Similarly, bidder 2 would receive two goods and pay 13 for the first and 11 for the second good and receive a payoff of $16 + 14 - 13 - 11 = 6$. Bidder 3 pays 13 for the single good awarded, yielding a payoff of $15 - 13 = 2$. In sum, in the multi unit Vickrey auction format, the seller would receive a revenue of $13 + 13 + 12 + 11 + 10 = 59$.

As it can be seen from the example, given identical bid vectors in all of the three distinctive auction formats, the seller receives most in the multi unit discriminatory auction (85), followed by the multi unit uniform price auction (65) and the multi unit Vickrey auction (59). From the perspective of the bidders however, the multi unit Vickrey auction is most favourable since it yields the highest payoffs for all three bidders, followed by the multi unit uniform price auction and the multi unit discriminatory auction where they all receive payoffs of zero.[33] The outcome of all three auction formats in this simplified example however is efficient. It has thereby been illustrated that the choice of an auction design given multiple homogeneous goods are at sale can have contrary benefits for the seller and the bidders. Note however that based on the results of this example, universal predictions on the superiority of one of the different auction formats cannot be made on no account.

All of the multi unit auctions described so far are sealed bid in that every bidder submits a secret bid to the seller, indicating how much he is willing to pay for each unit. Recall that if all bidders only request one unit, even for the case of multi unit supply, the Revenue Equivalence Theorem can be applied, that is, the multi

[30] See Vickrey (1961).

[31] Note however that when only a single good was sold, the multi unit Vickrey auction reduces to a single unit second price sealed bid auction identical to the multi unit uniform price auction.

[32] The proof provided by Vickrey (1961) and in section 5.2 of this thesis can accordingly be extended to the case for multi unit demand since by not bidding truthfully for all units, a bidder would forgo surplus with positive probability.

[33] Note that in the example bidder 3 is however indifferent between the multi unit uniform price auction and the multi unit Vickrey auction.

unit discriminatory auction, the multi unit uniform price auction as well as the multi unit Vickrey auction all generate the same expected revenue. However, as it has been highlighted, with multi unit demand in general only the Vickrey auction is efficient and therefore the Revenue Equivalence Theorem cannot universally be applied to multi unit auctions. In fact, even an exclusive ranking of the different auction formats in terms of seller revenue cannot be obtained since the auction outcome critically depends on the distribution of bidder valuations.

Alternatively to the sealed bid formats discussed so far, the seller can just as well correspondingly choose open auction formats to tender his goods:

In *multi unit open descending price auctions* or multi unit Dutch auctions, the price is continuously being lowered by the seller from a level high enough where no bidder is willing to buy any good until bidders will eventually agree to buy, similar to the single unit descending price auction. The goods are then sold at the prevailing prices and the auction ends when all goods are sold. The outcome of the multi unit descending price auction is equivalent to the multi unit discriminatory auction.[34]

In contrast, when the seller chooses a *multi unit open ascending price auction* or multi unit English auction, the auction starts at a relatively low price, where aggregate demand exceeds supply, that is subsequently being gradually raised. As the price increases, the demand at the prevailing price is being reduced and the auction ends when aggregate demand equals supply. The goods are then being sold to all remaining bidders at that specific price, therefore leading to the same outcome as the multi unit uniform price auction.[35]

Finally, the *multi unit Ausubel auction* is a modified multi unit ascending price auction format.[36] As in the multi unit open ascending price auction, the seller starts with a relatively low price where aggregate demand exceeds supply that is then being continuously raised. As the price increases, bidders will eventually reduce their amount requested and the goods are sold to those bidders that face a positive residual supply at the prevailing price. As it is the case in the multi unit Vickrey auction, it is a dominant strategy to bid truthfully resulting in an efficient allocation. Therefore, the multi unit Ausubel auction is outcome equivalent to the multi unit Vickrey auction.

[34] If only a single good was at sale, the multi unit open descending price auction is equivalent to a single unit first price sealed bid auction.

[35] Clearly, when offering only a single good, the multi unit open ascending price auction reduces to a standard single unit ascending price auction.

[36] See Ausubel (1997).

Note that under the assumption of independent private valuations, any equilibrium in the multi unit discriminatory auction is equivalent to an equilibrium in the multi unit open descending price auction, any equilibrium in the multi unit uniform price auction is equivalent to the multi unit open ascending price auction and any equilibrium in the multi unit Vickrey auction to one in the multi unit Ausubel auction.[37]

### Sequential Auctions

Alternatively to simultaneously selling multiple identical units in a single auction, sellers can choose to offer the goods in separate sequential auctions. In the following, for simplicity, it is assumed that bidders all have single unit demand, that is, they only bid for up to a single good in the respective auctions.[38]

If the seller chooses to tender the goods in *sequential first price auctions*, the goods are sold in order of decreasing bids, that is, the respective goods will be allocated to the bidder with the highest bid in each of the auctions and successful bidders pay the price they bid. Since the goods are sold in a sequence of auctions, unsuccessful bidders can opt to bid for a good in any subsequent auction. Note that even if all prices of earlier auctions are disclosed, this has no impact on bidder behaviour since valuations are private and independently and identically distributed. Therefore, optimal bidder behaviour in late auctions does not depend on the prices determined by already completed auctions. However, if bidders fail to win a good in one auction, they tend to bid higher in subsequent auctions since the number of goods still for sale decreases with the number of completed auctions. At the same time, since the number of remaining bidders decreases with every auction ended, as a consequence, the valuations of the remaining bidders are smaller which induces bidders to reduce their bids accordingly. Indeed, in equilibrium these two contrary effects entirely offset each other leading to a martingale price path, that is, the expected price of an auction equals the price paid in the antecedent auction and the goods are being allocated efficiently.[39]

If the goods were instead sold by *sequential second price auctions*, it only is a dominant strategy to bid truthfully in the last auction to be held. In earlier auctions however, bidders optimally submit bids lower than their valuations since there is potential additional value from higher expected payoff by winning a good in some

[37] See Krishna (2002), pp. 179-181.
[38] For the case of multi unit demand, see Weber (1983), Hausch (1986), Black and de Meza (1992) and Milgrom and Weber (2000).
[39] See Milgrom and Weber (2000).

later auction. As in sequential first price auctions, the equilibrium price path is a martingale.[40] As for the case of sequential first price auctions, sequential second price auctions allocate the goods efficiently among all participating bidders, despite the fact that bidders bid more aggressively in sequential second price auctions. As a result, the two auction formats are revenue equivalent.[41]

## 2.6 Reserve Prices

This subsection aims to resume the key findings of the impact of reserve prices in auctions presented by auction theory. The different methods to sell a good by way of an auction discussed so far can be enhanced by reserve prices, permitting the seller to strategically and more actively engage in the auction mechanism instead of leaving the determination of the final price only to the competing bidders.

A reserve price in an auction specifies a minimum bidding level required from the bidders to successfully participate, that is, only bids that at least meet the reserve price are deemed adequate by the seller. An appropriately set reserve price may prevent the sale of the good at price levels regarded insufficient by the seller and can foster competition amongst bidders. When choosing to augment an auction by a reserve price, the seller can choose between either a *public reserve price* that is fully disclosed before the auction takes place and can thereafter not be altered or a *secret reserve price* that is not being publicly announced prior to the beginning of the auction process. As for public reserve prices, given an auction has a secret reserve price, the good is only sold if the final highest bid is above the reserve price. Clearly, a seller would optimally choose a reserve price at least as high as his own valuation of the good since otherwise he would run the risk that the transaction takes place at a price that is lower than his valuation, ultimately yielding a loss for the seller with positive probability (that is, if the final price is between the reserve price and the seller's actual valuation). As it will be shown hereafter, given specific circumstances, a seller might however optimally choose a reserve price that strictly exceeds his own valuation for the good.

Given independent private bidder valuations, a seller may increase his expected revenue by appropriately setting a reserve price.[42] If he chooses a reserve price that is able to extract (some of) the highest bidder's surplus, it is clearly beneficial for the seller to choose a reserve price that is well above his own valuation. However,

[40] See Milgrom and Weber (2000).
[41] See Krishna (2002), p. 217.
[42] See Myerson (1981) and Reiley and Samuelson (1981).

since bidder valuations are private information, the choice of a reserve price above the seller's valuation may at the same time lead to an ex post inefficient auction outcome when the highest bidder's valuation is below the posted reserve price but at the same time exceeding the seller's own valuation of the good, implying that the good will not be sold when instead it would have been sold if the reserve price would have been set at some lower level still exceeding the seller's valuation for the good. Consider first the case of a single unit second price sealed bid auction: A reserve price in that case has no effect on bidder behaviour since it is still a dominant strategy to bid truthfully. If the seller chooses a reserve price that strictly exceeds the second highest bidder's valuation but is below or at most equal to the highest bidder's valuation, the transaction takes place at a price higher as if no reserve price was given since then, the final price would have been equal to the second highest bidder's valuation and thus lower than the reserve price. In that case, the seller would increase his revenue versus the auction without a reserve price. Next, consider a single unit first price sealed bid auction that is enhanced by a reserve price. Analogously to the second price sealed bid auction, besides the fact that a reserve price strictly above zero may exclude some bidders with relatively low valuations, it may raise more seller revenue by inducing the remaining bidders to bid more aggressively.[43]

Similarly, when bidder valuations instead were interdependent, a reserve price will usually increase expected seller revenue when chosen appropriately.[44] As bidders in this case do not know the value of the good with certainty, the disclosed level of a reserve price may reveal information on the effective value of the good that allows bidders to amend their bidding behaviour. An obvious consequence is that they reduce bid shading and thereby leading to higher price levels. On the other hand however, given positive reserve prices, bidders with valuations less than the reserve price who would otherwise have participated in the auction are being excluded leading to a potential informational loss for the remaining bidders since they can no longer observe actions taken by those bidders.[45] Thus, even though reserve prices may in general strictly increase expected seller revenue, their virtues critically depend on their respective level chosen by the seller.

[43] See Krishna (2002), pp. 24-26 and Menezes and Monteiro (2005), pp. 22-24.
[44] See Milgrom and Weber (1982).
[45] This is particularly the case in open ascending price auctions.

## 2.7 Strategic Manipulation in Auctions

Despite the eminent advantages of auctions by virtue of its price discovery ability in comparison to fixed price markets, the existence of asymmetric information in auctions can give rise to several anomalies that may in part deteriorate its evident merits. Recent literature has pointed to increasing fraudulent and illicit behaviour on both the seller and the bidder side, especially with regard to online auctions. On numerous markets where auction mechanisms are applied, rules or regulations against fraud are difficult to enforce due to the fact that auction participants can oftentimes hide themselves behind artificial identities.[46] Amongst the most notable kinds of strategic manipulation are collusion, shilling, bid shielding and bid sniping.[47]

### Collusion

A major concern sellers may face is the risk of collusion amongst bidders.[48] When relaxing the assumption of bidders independently and noncooperatively making their bidding decisions, collusion among some or all of the bidders may arise with the ultimate aim to avoid bidding up prices. The possibility of collusion will in the following be presented for the case of single unit first and second price auctions. Similarly, multi unit auctions are also suspectible to collusion.[49]

To illustrate the phenomenon of collusion, assume again the independent private values model, that is, bidders' valuations are private and independently drawn from an identical distribution. Assume further that bidders are ex ante asymmetric.[50] Start by considering a single unit second price sealed bid auction where a subset from all participating bidders colludes.[51] For those bidders colluding it is straightforward that bidding truthfully remains to be a dominant strategy as it is for bidders outside the bidding ring. It is moreover a dominant strategy for such a bidding ring as a whole to only submit a single bid equal to its highest bidder's valuation while for all

[46] Online auction platforms increasingly try to eliminate strategic bid manipulation by requiring information that identifies its customers, such as details on their credit cards or residential authentification.
[47] See Lucking-Reiley (2000).
[48] Collusive arrangements in auctions are alternatively called "bidding rings" or "cartels" in existing literature (see McAffee and McMillan (1992)). These different notations are in the following being synonymously used.
[49] See Cramton and Schwartz (2000), Bajari and Summers (2002), Brusco and Lopomo (2002), Klemperer (2004) and Milgrom (2006).
[50] Even if bidders were ex ante symmetric, the presence of collusion would naturally introduce asymmetries between bidders who actively collude and bidders who do not. See Krishna (2002), p. 152.
[51] See Robinson (1985), Graham and Marshall (1987) and Pesendorfer and Swinkels (2000).

other bidders submitting bids of zero - or at most equal to the reserve price.[52] As a result, only a single "serious" bid is being submitted by the bidding ring, leading to possible gains for its bidders by suppressing competition.[53] Given that with positive probability a colluding bidder has the second highest bidder's valuation over the entire bidder population (including both bidders inside and outside of the ring), the final price paid by the bidding ring may turn out to be strictly lower than without such a collusive agreement. Therefore, engaging in collusion may be profitable for its members since overall, the expected prices paid by ring members are lower versus an auction in which they did not collude. Further, note that all other bidders who are not part of the bidding ring are not worse off since their probability of winning the good as well as their expected payoff remain the same. Since thereby the expected profit of noncolluding bidders is unaffected, the gains from collusion are at the full expense of seller revenue. Obviously, from the bidders' perspective, a bidding ring including all bidders is most efficient since thereby their overall profits are maximized. As a consequence, second price auctions are remarkably vulnerable to collusion.[54]

Next, consider a single unit first price sealed bid auction.[55] Again, when bidders collude, they would seek for the lowest possible price necessary to obtain the good. However, unlike in a second price sealed bid auction, bidders effectively colluding have an incentive to cheat on the agreement: If a bidder indeed had a valuation above the price agreed upon by the bidding ring, he could strictly increase his expected payoff by defeating the gains from collusion if he submitted a bid exceeding the price optimal for the bidding ring. As an example, examine the case where all bidders in the auction would collude. Then, it would be in the bidding ring's best interest to submit a price equal to the reserve price or alternatively the lowest necessary price to obtain the good. If a bidder had a valuation above the reserve price - or the price required for successful bidding, he would have an incentive to submit a bid exceeding a price agreed upon, thereby winning the good and at the same time gain a profit exceeding his expected payoff he would obtain when colluding with

[52] Note that the bidding ring has to induce bidders to fully reveal their valuations to facilitate an effective operation. Therefore, an optimal mechanism design needs to be implemented amongst the colluding bidders for them to truthfully reveal this information. At this point however, it is being abstained from introducing more complexity and it is simply assumed that bidding rings indeed work in the way described. For a more detailed overview on mechanism design, see Myerson (1981) and Krishna (2002).

[53] Obviously, these gains must be shared amongst the members of the ring to offer incentives to collude.

[54] See Robinson (1985).

[55] See McAfee and McMillan (1992).

positive probability.[56] Therefore, in contrast to second price sealed bid auctions, first price sealed auctions usually are more robust to collusion. Moreover, collusion in first price auctions has an impact on overall bidder behaviour versus second price auctions where it remains unaffected. Specifically, non-colluding bidders in first price auctions would have an incentive to accordingly lower their bids when presuming that collusion exists in an auction. Again, as for the case of a second price sealed bid auction, collusion can potentially reduce seller revenue.

### Shilling

A phenomenon that has attracted considerable attention in recent auction theory is the possibility of sellers to manipulate the auction outcome by shilling.[57] Shilling is referred to events where a seller tries to drive the final auction price higher when only a single bidder remains in the auction by bidding against that bidder himself.[58] Even if in most auctions sellers are not allowed to submit bids in their own auction, they can circumvent this restriction by either creating alias identities and then submitting bids or by instructing other bidders to submit bids on their behalf.[59] Thereby, the final auction price may be higher than in the auction without shilling and the winning bidder ends up paying more with positive probability. Shilling could therefore be regarded as secret and dynamic reserve prices that are not publicly announced and are being modified by the seller in the course of an auction. In the context of independent private bidder valuations it is however not at all times optimal for the seller to shill bids since it could enforce bid shading if bidders anticipate that the seller will manipulate the auction.[60] Likewise, even in a common values setting it might not always be appropriate to shill bids when bidders anticipate shilling.

### Bid Shielding

Bid shielding occurs when a bidder submits a relatively low bid for the good at sale and at the same time illicitly submits an extremely high bid (again by either another alias identity or getting another bidder to submit a bid on his behalf).[61]

[56] Note however that specific forms of enforcement to behave appropriately within the bidding ring could be introduced.
[57] See Sinha and Greenleaf (2000), Chakraborty and Kosmopoulou (2004), Engelberg and Williams (2005) and Kauffman and Wood (2005).
[58] A seller could similarly bid against numerous active bidders.
[59] This phenomenon is especially common on online auction platforms. The possibility to use different anonymous identities on such auction platforms substantially facilitates shilling.
[60] See Sinha and Greenleaf (2000).
[61] See Lucking-Reiley (2000).

Thereby, the high bid acts as a shield for such a bidder since the high bid excludes other bidders from participating in the auction if the current bid already exceeds their valuation of the good. The high bid is then being retracted just before the end of the auction and leaving the lower bid possibly winning the auction despite the fact that other bidders would have participated in the auction when the lower of the two bids was the current bid.[62] Bid shielding is therefore only effective in open ascending price auctions.

### Bid Sniping

A further phenomenon observed on various auction markets is bid sniping.[63] Bid sniping is identified as the event of high volume bidding near the end of an auction. Arguing solely with auction theory, at first sight such late bidding should not have a significant impact on the outcome of an auction if bidders condition their bidding behaviour primarily on their individual valuations for the good, especially in independent values auctions. Specifically, in sealed bid auctions such late bidding does certainly not have any impact on bidder behaviour and the outcome of the auction. However, in open ascending price auctions where information is being revealed during the auction, it may be reasonable for bidders to snipe, especially if bidders do not initially bid according to their valuation and increment their bids during the course of the auction. In this case, bidding late in an auction can be optimal given a predetermined closing time of the auction if a bidder submits a relatively high bid to which other bidders cannot respond in time, that is, before the auction ends.[64] Thus, by optimally submitting a late bid, such a bidder could outbid the incremental bidder and win the auction at a comparably low price which in turn reduces seller revenue with positive probability.

Obviously, given interdependent private bidder valuations, late bidding is reasonable since bidders continuously update their valuations given the continuous flow of information observed during the auction.[65] Furthermore, with interdependent private valuations, it can be in an individual bidder's best interest not to reveal information on his valuation in an early stage of the auction that would induce other bidders to

[62] Bid shielding can only emerge at very specific auction mechanisms where bidders can indeed observe the bids and enter the auction at any given time of the auction. Another restriction is that the auction end must be known ex ante for the shielding bidder to be able to manipulate and bidders must be able to withdraw their bids.

[63] See Ockenfels and Roth (2002), Bajari and Hortascu (2003), Hayne and Vijayasarathy (2003) and Anwar et al. (2006).

[64] See Ockenfels and Roth (2006).

[65] See Bajari and Hortascu (2003), Ockenfels and Roth (2006) and Rasmusen (2006).

update their valuations and thereby lessen the probability of their successful participation in the auction (and receiving the good at sale at a relatively low price). With regard to simultaneous auctions of identical goods, the possibility of observing the development of prices over a number of distinct auctions may further create incentives for bid sniping in that bidders can dynamically update their bids in the different auctions, and at any one time competitively submit a bid at the auction with the lowest current price.[66] A possibility for the seller to reduce the negative impact of bid sniping on the auction outcome is to offer extension periods that allow auctions with a predetermined ending date to continue for some time given that bids have been submitted shortly before an auction was scheduled to end.

[66] See Stryszowska (2005), Anwar et al. (2006), and Peters and Severinov (2006).

# Chapter 3

# Buyout Options in Auctions

## 3.1 Overview

The diffusion of the internet and online trading engines has brought forward a renewed interest in auction design and its implications for participating agents. The increasing number of items sold on platforms such as eBay, Amazon and similar suppliers of online trading systems asserts the importance of suitable auction mechanisms that may increase seller revenue, optimally allocate goods amongst bidders and foster market efficiency. The current stance of auction literature offers a broad range of theoretical and empirical work that addresses and analyzes a multitude of topics. The aim of this chapter is to give a modest overview on the topic of Buyout Options in auctions and its present-day use followed by a survey of the existing literature in the subsequent chapter.

Before describing the characteristics of Buyout Options in auctions, a clarification due to its terminology is necessary to avoid potential misunderstandings since Buyout Options in auctions do not feature the usual apparent attributes that can be found at options on financial markets. In financial market theory, an option is usually referred to as a contract that gives the holder the right, but not the obligation, to close a certain transaction at some specific time in the future. In contrast, the option of interest here does not comprise a similar contract but can rather be seen as the right to conduct a transaction immediately without the option of the execution at some later time, as will be described in the following paragraphs. In other words, Buyout Options in auctions do rather resemble offer prices than any future contract.

Buyout Prices in auctions, though their names and rules may vary in detail, are prices for the good at sale that are ex ante announced by the seller, the existence and amount of which are publicly known by all auction participants, i.e., all participating bidders and sellers. A Buyout Option allows the seller to set a predetermined price at which any bidder may immediately purchase the item without the necessity of competing in the auction or even before the auction effectively starts. As long as bidding in the auction is less than the announced Buyout Price, the auction continues until no bidder is willing to increase his current bid (there can exist events during an auction where the Buyout Option might disappear though). If a bidder is however willing to post a bid equal to the Buyout Price, the auction immediately ends and the good is sold to that specific bidder at the posted Buyout Price. Thereby, Buyout Options allow bidders to buy an early end to the auction when exercising the option.[1] The determination of the final price in an auction enhanced with a Buyout Option though critically depends on the specific auction mechanism in place, that is, if the Buyout Option is indeed being exercised and if so, under what circumstances or if the good is sold by way of the standard auction. According to this definition, a Buyout Price is in some sense the functional opposite of a reserve price in that a reserve price defines a minimum bidding level required to successfully participate in an auction while a Buyout Price establishes a maximum bidding level at which the seller is willing to tender the good immediately.[2]

There are different forms of Buyout Options conceivable and indeed already in use on several auction markets. They can be distinctively classified into *permanent*, *temporary* and *limited* Buyout Options.

*Permanent* Buyout Options remain available throughout the entire course of an auction and do not disappear as soon as a prespecified event occurs short of being exercised by any bidder. Thus, bidders cannot alter the availability of a permanent Buyout Option except by its execution or alternatively by submitting a bid higher than the prevailing Buyout Price.[3]

On the other hand, a *temporary* Buyout Option disappears as soon as any bid is being submitted at or above a given (secret) reserve price. Clearly, when a seller offers a temporary Buyout Option, bidders must decide on whether to exercise the Buyout Option before the actual auction starts whereas when offering a permanent Buyout Option, bidders can wait and observe the process of the auction before

[1] See Lucking-Reiley (2000).

[2] The definition of Buyout Options in auctions given here is not comprehensive in that they can also be implemented in multi unit or share auctions, as will be shown later. For simplicity, the Buyout Option here and in the following paragraphs is exemplified in the forefront of single unit auctions despite the fact that any arbitrary auction mechanism can be enhanced by such an option.

[3] Submitting a bid above the Buyout Price is however not reasonable since then such a bidder would pay a price higher than the minimum bidding level required to receive the item.

exercising the option so long as no other bidder exercises the option or submits a bid above the Buyout Price. Thus, if the auction is open and bidders are able to observe their competitors' bids, they may find a permanent Buyout Option beneficial since in that case they can condition their decision on the execution of the option on the other bidders' behaviour. However, if the auction is sealed, that is, if bidders cannot observe their competitors' bids, then a permanent Buyout Option does not offer any advantages versus a temporary Buyout Option since bidders cannot condition their bidding behaviour for the Buyout Option on the other players' bids. Temporary Buyout Options do therefore not only disappear as soon as any appropriate bid has been submitted but also endow individual bidders with substantial control over the availability of the option for other bidders by either exercising the option or simply submitting an appropriate bid whereas in the case of permanent Buyout Options, bidders can only eliminate the option by either exercising it or posting a bid above the given Buyout Price.

Buyout Prices are considered to be *limited* if they are only in effect for a limited period of time during an auction and cease to be available after a specific point in time or if a particular event occurs. Several kinds of limited Buyout Options are conceivable, despite not yet systematically discussed in existing literature. The time of the disappearance of a limited Buyout Option could for instance either be ex ante fixed by the seller or be conditioned on bidding behaviour in the auction. If the Buyout Option however ceases to be available at a prespecified time in the auction, individual bidders cannot influence its disappearance other than exercising it whereas if it is conditioned on bidding behaviour, they have some control. The fundamental discrepancies in the duration of the availability of the different Buyout Options can subsequently lead to differing auction equilibria and results.[4]

Buyout Options can further be classified as *static* and *dynamic* Buyout Prices. *Static* Buyout Prices are constant over time and do not change during the progression of an auction. *Dynamic* Buyout Prices contrariwise are not constant during the course of an auction but may instead vary over time according to a prespecified trajectory or if certain events occur. The practice of static Buyout Prices seems to be predominantly in use on today's auction markets and dynamic Buyout Prices have so far only been discussed very scarcely in existing literature. Furthermore, the greater complexity of dynamic Buyout Prices eclipses its practicability for a broader use in real world auctions.

[4] See Lee and Ahn (2004), Reynolds and Wooders (2005) and Gupta and Gallien (2007).

As it has been described beforehand, the enhancement of an auction by a Buyout Option offers a unique hybrid market institution, combining both fixed price offers and auction mechanisms. The Buyout Option thus allows a seller to offer the good in a hybrid of auction and posted prices instead of exclusively choosing either of the two institutions which under certain circumstances might be beneficial for the auction participants. The common view on Buyout Options in the current theoretical literature is that they can be viewed as providing a form of insurance for risk averse or time impatient auction participants.[5] The seller can potentially increase expected revenue by exploiting the risk aversion of bidders if their valuations indeed exceed a properly set Buyout Price. Additionally, a risk averse seller can further benefit from the implementation of a Buyout Option since it may reduce the variance of his revenue. Risk averse bidders on the other hand are generally not worse off when such options are being offered since they can exercise the Buyout Option to be able to achieve a positive profit instead of bearing the risk of not winning the auction and are thus willing to pay a risk premium on the allocation of the good.

The option allowing a potential bidder to purchase the item being auctioned at a prespecified Buyout Price instead of attempting to obtain the good through a traditional auction procedure does therefore feature several benefits for the bidders. Amongst others, the execution of such an option allows a bidder to:[6] (i) Get the good right away (allowing the bidder to reduce costs associated with waiting for the auction to close, which could be at some indefinite time in the future), (ii) secure an ex ante specified price (traditional bidding in an auction may be able to drive the final selling price above the Buyout Price), (iii) eliminate any risk of losing the auction to a bidder with a higher willingness to pay and (iv) save monitoring costs associated with ongoing screening of the process of the auction.

The decision on whether to execute the Buyout Option when available or bidding in the auction instead however is a trade-off for bidders: If a bidder exercises the Buyout Option he could buy himself an early end of the auction and receiving the good at the ex ante specified price and guaranteeing himself a positive profit. A bidder may potentially pay less when exercising the Buyout Option than he would have needed to when bidding in a traditional auction. At the same time though he would run the risk of forfeiting additional profit he could have gained when instead only participating in the traditional auction if the final price of the good would have been lower than the Buyout Price.

[5] See Hidvégi et al. (2006).
[6] See Mathews (2004).

When a Buyout Option is exercised by a bidder, it may benefit both the bidder and the seller by terminating the auction earlier than if no such option would have been in place. Clearly, when auction participants are time impatient or the auction process itself is costly, than an early end of the auction is beneficial. Despite these possible obvious positive effects of an early termination of the auction, Buyout Options might at the same time adversely affect seller revenue since bidders that would have participated in the auction without a Buyout Option may be ruled out due to the early termination of the auction. Thus, when the seller chooses to offer a Buyout Option which in turn is exercised with positive probability before the regular end of the auction, he may prevent bidders with relatively high valuations to bid for the good and thereby lower his revenue.

When enhancing an auction by a Buyout Option, the seller can guarantee himself a price for the good at sale which exceeds the reserve price with positive probability. However, if the Buyout Option is being exercised by bidders, the seller could run the risk of losing potential revenue since bidders with valuations exceeding the Buyout Price that indeed exercise the Buyout Option would also potentially have paid more in a traditional auction than they do when exercising the Buyout Option. Furthermore, when bidders randomly arrive at the auction, potential bidders have a greater likelihood of viewing the good at sale if the good is indeed posted for a longer time instead of only a very short time.[7] Therefore, intuitively the striking question of whether such an option is in fact not reducing the seller's expected revenue does arise and if it actually makes sense that a seller limits the range of possible prices in an auction at an ex ante specified upper bound and if so, how this upper bound should be optimally set. Moreover, Buyout Options do, when indeed being exercised, defy the benefits of auctions in that they eliminate the dynamic pricing mechanism that is being credited with the merit of ultimately being an efficient way of economic transactions.

As it has been remarked, when Buyout Options are in place, besides the fact that the seller has to bear some loss of revenue, the final outcome of an auction is inefficient with positive probability. Thus, special attention must be turned to the possibility of allocative inefficiency that may be induced by the enhancement of an auction by such an option. A Buyout Option allows any given bidder to buy the good immediately and thereby ruling out all other bidders. Suppose the Buyout Option is being exercised by a bidder whose valuation for the good is not the highest among all bidders participating in the auction. Then, the outcome of the auction is

[7] See McAfee and McMillan (1987), Gilkeson and Reynolds (2003) and Song and Baker (2007).

inefficient, since the bidder who most values the good would not win. If the seller had instead chosen not to offer a Buyout Option, the auction would have been won by the bidder with the highest valuation which would have been an efficient result. Thus, by facilitating inefficient outcomes, Buyout Options may purge one of the key advantages of auction mechanisms, that is, allocative efficiency.

Buyout Options however may reduce the probability of collusion amongst bidders by creating incentives for bidders to cheat on any agreement when their payoff from indeed exercising the option strictly exceeds the gains they could obtain by colluding. Similarly, Buyout Options may diminish bid shielding because illicitly submitting high bids becomes costly (clearly, such deceptive bids should not exceed the Buyout Price). Bidders that could have been ruled out when bid shields were present can then still successfully participate in an auction. In addition, bid sniping may be eliminated when an auction is ended at an early stage when the Buyout Option is being executed before the predetermined conclusion of the bidding period. While the enhancement of an auction by a Buyout Option in respect of the possibility of strategic manipulation on the bidders' side is clearly in favour of the seller, bidders may likewise benefit from such options. From the bidders' perspective, an auction enhanced by a Buyout Option may likewise reduce their risk of manipulative exposure since it allows them to elude shilling by exercising the option. These additional benefits on either side of the transaction would however require the seller to accordingly choose the Buyout Price. Thus, besides the beforehand described obvious advantages of Buyout Options, they may as well contribute to the reduction of strategic manipulation in auctions.

The potential benefits and hazards described above indicate the intriguing complexity of whether Buyout Options are indeed beneficial for the auction participants and if so, how a seller would optimally choose its price. The analysis of Buyout Options in auctions is therefore not only mathematically greatly challenging but as well intuitively intricate.

## 3.2 Numerical Example

To illustrate the complex and possibly diametrically opposed benefits of Buyout Options in auctions described above, a simple numerical example of an auction enhanced by a temporary Buyout Option is being presented hereafter.[8]

[8] The possibility to post a reserve price is omitted to be able to solely illustrate the impacts the seller's choice of Buyout Prices can have on the outcome of the auction.

Assume that a seller offers a single good for sale in a standard second price sealed bid auction. Further, assume that there are three bidders (denoted $A$, $B$ and $C$) with individual private valuations of $v_A = 10$, $v_B = 7$ and $v_C = 3$, where $v_i$ denotes the valuation of bidder $i$, each of them willing to acquire the good. Suppose for simplicity that the seller's aim is solely to maximize his expected revenue and that individual bidders' payoffs are $\pi_i = v_i - p$, where $p$ denotes the price paid if they indeed win the auction, or zero when they do not successfully participate.

If the seller were not able to offer a Buyout Option, he would simply sell the good by way of a standard second price sealed bid auction mechanism. Recall that in a second price sealed bid auction it is a dominant strategy for every bidder to truthfully bid his valuation. Then, bidder $A$ would win the auction and pay a price $p = 7$.[9] Therefore, the seller receives a revenue of 7 and bidder $A$'s payoff is $\pi_A = 10 - 7 = 3$. Note that the auction outcome is efficient in that the bidder who values the good most gets it.

Now consider the case where the seller is able to offer a Buyout Option at price $b$. The Buyout Option is only available so long as no traditional bid has been submitted, that is, before the auction starts, and disappears if no bidder chooses to exercise it before the commencement of the actual auction. To keep things simple, it is assumed that any bidder with a valuation above or equal to the Buyout Price will exercise the Buyout Option.[10] Furthermore, if two or more bidders exercise the Buyout Option, the good is randomly assigned to one of these bidders with the same probability. Thus, if two bidders exercise the Buyout Option, they both receive the good with probability $\frac{1}{2}$ and when all three bidders exercise the option, they each receive it with probability $\frac{1}{3}$.

Obviously, a seller would not want to offer a Buyout Price $b = 0$ since this would amount to offering the good for free. If the seller chooses a Buyout Price $0 < b_\alpha \leq 3$, all three bidders would exercise the option and the seller receives $b_\alpha \leq 3$. If bidder $A$ receives the item, his payoff is $\pi_A = 10 - b_\alpha$, if $B$ wins, his payoff is $\pi_A = 7 - b_\alpha$ and if $C$ is the winner, he receives $\pi_C = 3 - b_\alpha$. If the seller would instead choose a Buyout Price $3 < b_\beta \leq 7$, bidder $A$ and $B$ would exercise the Buyout Option, each of them receiving the good with probability $\frac{1}{2}$ and gaining an expected payoff

[9] Since in a second price sealed bid auction the bidder submitting the highest bid wins the auction and ends up paying the second highest bid submitted.

[10] This assumption is restrictive but helps to make the results and its implications tractable in this numerical example. As it will be shown later in the analysis of the model in chapter 5, it is much closer to reality to specify some threshold value depending on individual bidder valuations that determines optimal individual bidder behaviour.

of $\pi_i = \frac{1}{2}(v_i - b_\beta)$ (and the seller receives $b_\beta$). Note that the auction outcome is inefficient with positive probability so long as the seller chooses a Buyout Price $b \leq 7$ since the successful bidder is randomly chosen and the bidder with the highest valuation might not win the auction.

Next, consider the case where the seller sets the Buyout Price $7 < b_\gamma \leq 10$. Then, only bidder $A$ will exercise the Buyout Option and receive a payoff of $\pi_A = 10 - b_\gamma$ with probability one while the seller obtains $b_\gamma$. In this case, the allocation of the good is again efficient. If the seller would choose a Buyout Price exceeding the highest bidder's valuation ($b_\delta > 10$), no bidder will exercise the Buyout Option and the good is sold by a standard second price sealed bid auction with again bidder $A$ receiving the good at price $p = 7$ and obtaining a payoff of $\pi_A = 3$.

The results conditional on the seller's choice of the level of the Buyout Price are summarized in Table 3.1.

Table 3.1: Auction participant's welfare conditional on the Buyout Price

| **b** | $\boldsymbol{\Pi}_{Seller}$ | $\boldsymbol{\pi}_A$ | $\boldsymbol{\pi}_B$ | $\boldsymbol{\pi}_C$ | **Efficiency** |
|---|---|---|---|---|---|
| – | 7 | $10 - 7 = 3$ | 0 | 0 | *given* |
| $0 < b_\alpha \leq 3$ | $b_\alpha$ | $\frac{1}{3}(10 - b_\alpha)$ | $\frac{1}{3}(7 - b_\alpha)$ | $\frac{1}{3}(3 - b_\alpha)$ | *uncertain* |
| $3 < b_\beta \leq 7$ | $b_\beta$ | $\frac{1}{2}(10 - b_\beta)$ | $\frac{1}{2}(7 - b_\beta)$ | 0 | *uncertain* |
| $7 < b_\gamma \leq 10$ | $b_\gamma$ | $10 - b_\gamma$ | 0 | 0 | *given* |
| $10 < b_\delta$ | 7 | $10 - 7 = 3$ | 0 | 0 | *given* |

As can be seen from Figure 3.1, the seller would maximize his revenue by choosing a Buyout Price equal to the highest bidder's valuation, that is $b_\gamma = 10$. The shaded area in Figure 3.1, depicting excess seller revenue versus the standard second price sealed bid auction, shows that the seller's revenue is strictly higher than in the standard second price sealed bid auction when he chooses a Buyout Price $b \in (7, 10]$.

This result is not surprising since extracting the highest bidder's entire rent would per definitionem maximize seller revenue. If the seller instead offered a Buyout Price lower than the second highest bidder's valuation, his revenue would be strictly lower versus the revenue he would have received when not offering a Buyout Option.

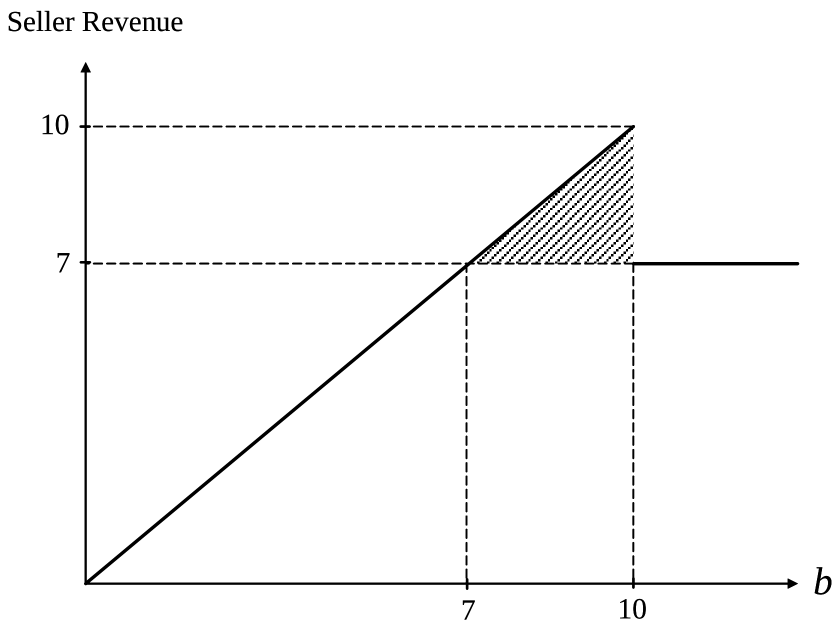


Figure 3.1: Seller revenue conditional on the Buyout Price

The seller would be indifferent between not offering a Buyout Option and setting a Buyout Price high enough so that no bidder will exercise the option ($b > 10$). The challenge however is that the seller does not know the individual bidders' valuations since they are private information. To enhance the auction with a Buyout Option would therefore only be beneficial to the seller if he chooses a Buyout Price exceeding the second highest bidder's valuation but at most equal to the highest bidder's valuation. Thereby it has been shown that on one hand the seller can strictly increase his revenue by enhancing the auction with an appropriately set Buyout Price while on the other hand losing some revenue when missing the appropriate level of the Buyout Price.

With regard to bidders' payoffs, only the case of the bidder with the highest valuation is of interest since all other bidders cannot be worse off when the seller offers a Buyout Option: If the seller did not enhance the auction by a Buyout Option, they would all receive a payoff of zero. However, when the seller chooses to offer a Buyout Option, their expected payoff may strictly rise to some positive amount. Therefore, it is always beneficial for these bidders when such an option is in place.

As depicted in Figure 3.2, if the seller sets a Buyout Price above the highest bidder's valuation ($b > 10$), bidder $A$ would receive the same expected payoff as in the standard second price sealed bid auction ($\pi_A = 3$). If the prevailing Buyout Price would be between the second highest bidder's and the highest bidder's valuation, $A$

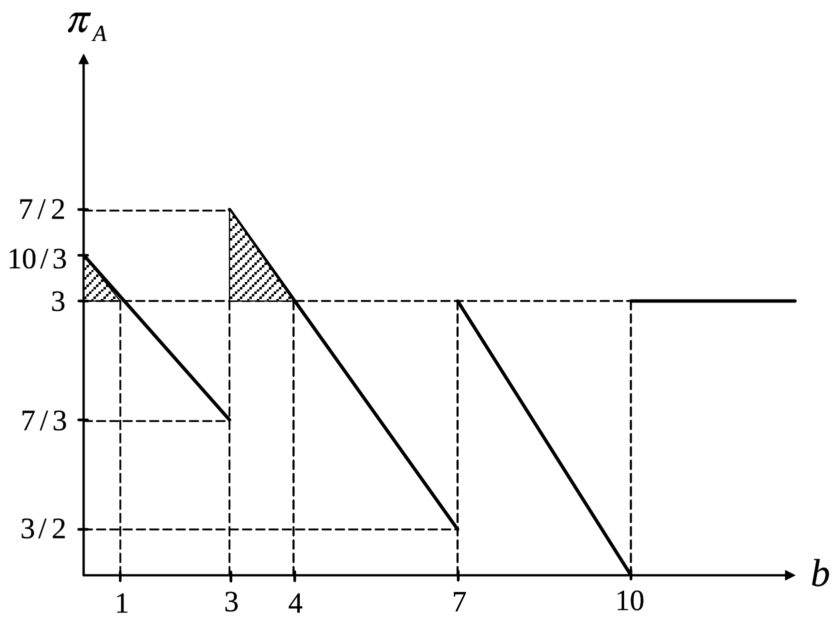


Figure 3.2: Bidder $A$'s revenue conditional on the Buyout Price

would be worse off since he exercises the Buyout Option and thereby reducing his payoff well below the level he would receive in the standard auction. However, if the seller would instead choose a Buyout Price of at most the second highest bidder's valuation ($b \leq 7$), bidder $A$ would be better off if and only if (i) the seller would choose a Buyout Price $b < 1$ if $b \in (0, 3]$ or (ii) the seller would choose a Buyout Price $b < 4$ if $b \in (3, 7]$.[11] The shaded areas in Figure 3.2 again identify the range of Buyout Prices that would be beneficial for $A$. Therefore, given the assumptions on bidder valuations, the bidder with the highest valuation would only prefer the auction enhanced by a Buyout Option if the seller sets relatively low Buyout Prices. It has therefore been shown that all bidders can cash in from the auction enhanced by a Buyout Option, while their individual benefits considerably depend on the price of the option chosen by the seller.

The numerical example presented here helps to emphasize the complexity of the problem faced by the seller as well as the possible impacts on both the allocative efficiency and bidder payoffs. Due to the prevalent informational asymmetry, both the seller and the bidders need to trade off the offering and execution respectively of the Buyout Option given their respective level of information. It has further been

[11] Ad (i): Clearly, bidder $A$ only has a higher expected payoff if $\frac{1}{3}(10 - b) > 3$ or by rearranging, $b < 1$ given that the seller chooses a Buyout Price $b \in (0, 3]$. Ad (ii): Analogously, bidder $A$ 's expected payoff is strictly higher in the setting with a Buyout Option if $\frac{1}{2}(10 - b) > 3$ or equivalently $b < 4$ for all $b \in (3, 7]$.

shown that it can be beneficial to both the seller and the bidders to enhance the auction by a Buyout Option. However, in the example, when the seller chooses a Buyout Price that strictly increases his revenue, the other side of the transaction is worse off since then the bidder exercising the Buyout Option has a strictly lower payoff than in the auction without a Buyout Option (while the other bidders' payoffs are unaffected). If the seller instead chooses a Buyout Price that is beneficial to at least one bidder, this would entail a detriment of his revenue. By this simple numerical example it has been shown while possibly favourable to one side of the transaction, the enhancement of the auction by a Buyout Option does not necessarily lead to a pareto improvement and might in addition lead to allocative inefficiency, which are both critical arguments when analyzing auction mechanisms.[12]

## 3.3 Empirical Evidence for the Use of Buyout Options in Auctions

After having introduced the diverse characteristics of Buyout Options, examples of actual implementations of such options in real world auctions are subsequently being presented to provide empirical evidence of its widespread use. The application of Buyout Options in online auctions has over the past few years extensively increased its share in market transactions and has become very popular with sellers.

As it is being revealed by data gathered from eBay's financial reports over the past few years, depicted in Figure 3.3, the share of transactions concluded by the execution of Buyout Options has experienced a steady increase since its introduction in the fourth quarter of the year 2000. According to eBay's quarterly financial statements, by the end of the second quarter of 2007, fixed price trading that can primarily be related to the "Buy It Now" feature on its worldwide trading platforms, accounted for 39% of total gross merchandise sales of USD 14.5 billion up from a share of approximately 11% of total gross merchandise sales of USD 5.25 billion in the second quarter of 2001.[13]

These data not only provide evidence that Buyout Options are indeed largely being used at auctions on eBay but also points to its economic relevance when considering the nominal value of goods sold by the execution of such options.

[12] Numerous more sophisticated models however may lead to a pareto improvement. See e.g. Mathews and Katzman (2006) and the model that will follow later on in chapter 5.

[13] Gross merchandise sales are the total value of all successfully closed items between users on eBay trading platforms. Further, note that Buyout Options were indeed already offered on approximately 35% of all listings on eBay in the second quarter of 2001.

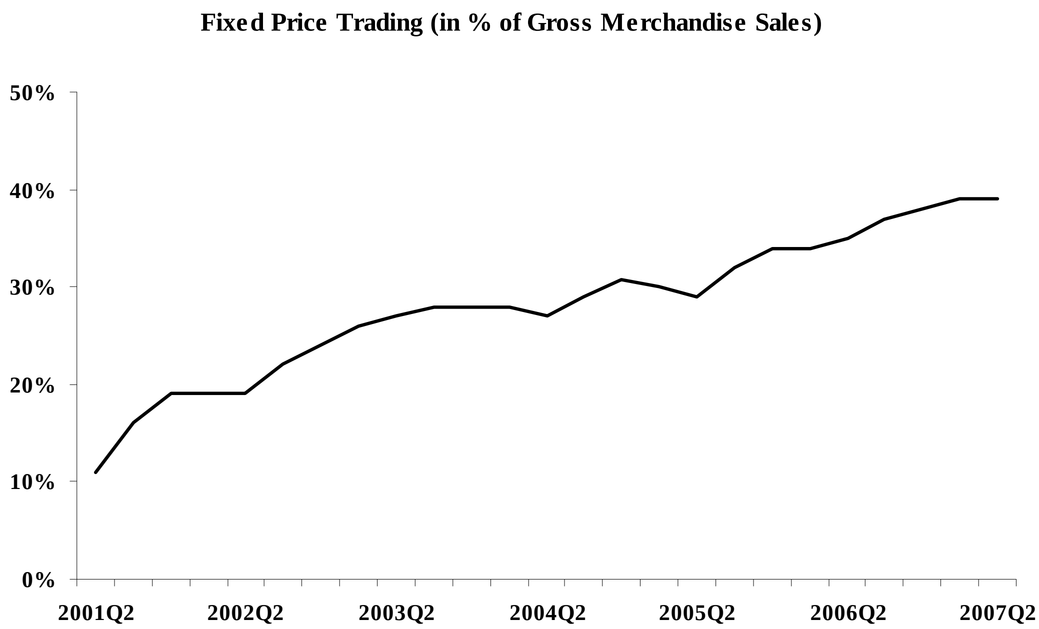


Figure 3.3: Fixed Price Trading on eBay, 2001Q2 - 2007Q2

In the following, a brief overview on some of today's most prominent online auction markets is given to exhibit the practical importance of Buyout Options in auction markets and the distinctive auction formats in use.

**eBay**

Today's market leader for online auctions, eBay, offers a range of auction formats to sell and buy items. These are: (i) Auction-like listings, including reserve price auctions and private auctions that follow a standard ascending price auction procedure, (ii) second chance offers, where sellers can under certain circumstances offer the good at sale to a bidder other than the auction's actual winner, (iii) multiple item auctions where a seller can simultaneously tender two or more identical goods and (iv) specialty sites, where amongst others online storefronts of high-volume sellers and vehicles listings can be found.[14] eBay offers two separate fixed price format features in auctions that allow potential buyers to get goods instantaneously at a posted price. The first feature is the "Buy It Now" option that enables bidders to purchase the item without placing a competitive bid. The "Buy It Now" option however disappears as soon as the reserve price for a good is met and is thereafter

[14] A comprehensive overview and description of these auction formats can be found on eBay's webpage, see www.ebay.com.

no longer available. It is therefore a temporary Buyout Option, as preliminary discussed. The other option offered is the "Best Offer" where instead of the seller, bidders are able to submit an offer specifying a price at which they are willing to buy the good immediately.[15] These offers can then be accepted at the discretion of the seller. Intuitively, these two features are very similar in that a fixed price offer is being made apart from traditional bidding in the auction by one side of the transaction that can then be accepted or forfeited by the other side.

There are three distinct ways to sell multiple identical items on eBay. Closest to the auction analyzed in the model of this thesis is the *online auction-type listing*. In an online auction-type listing the seller can simultaneously offer multiple identical goods in a standard multi unit uniform price auction.[16] Bidders can specify both the price they are willing to pay per unit and the number of goods they would like to acquire.[17] The winning bids at the end of the auction are determined on the basis of the highest bids per item in decreasing order and the price paid for all items is equal to the lowest winning bid. Thus, the price determination mechanism in this auction format is uniform in that the prices for every individual item are the same.[18] If several successful bids with the same price for an item are submitted, the bidders who submitted their bids earlier are awarded the goods. However, eBay's online auction-type listing differs in some critical ways from the model discussed in chapter 5: In the auction on eBay, bidders submit a price-quantity pair when participating in the auction while the model assumes single unit demand. Furthermore, the auction format employed in the model is a sealed bid auction while on eBay, it is an open multi unit uniform price auction where bidders can observe the bidding history during the auction and continuously alter their bidding behaviour (i.e., they can increase their bidding price when overbid by their respective competitors). However, this discrepancy can be disregarded for the case of multi unit open uniform price auctions, since in such an auction it is a dominant strategy for every bidder to truthfully bid up to his valuation so long as the final price is being determined by the highest losing bid.[19] With regard to the price determination mechanism, in the analysis of the model it is further assumed that price discrimination is adopted

[15] Best Offers automatically expire after 48 hours, making them limited Buyout Options with an ex ante know expiry time.

[16] eBay denotes these multi unit auctions "Dutch Auctions" in contrast to the common use of the terminology in auction theory.

[17] In multiple item listings, bidders cannot use proxy bidding to enter a maximum bid.

[18] It is possible that not all winning bidders receive the entire quantity they bid for, what is referred to as "bid rationing" in auction theory.

[19] In eBay's multiple online auction-type listings, winning bidders have further the right to refuse partial quantities. That is, if a bidder wins some but not the entire quantity of goods requested, he does not need to buy any of them and retract his bid.

whereas eBay uses a uniform price scheme.

When instead choosing to offer multiple goods by way of a *fixed price listing*, sellers simply list their goods at a fully disclosed ex ante determined fixed price. Potential buyers can then choose the quantity they want and pay the given price. A third way to sell multiple identical goods on eBay is by *lot listings* where the seller can choose to sell a well-defined number of goods as a "lot" to a single bidder. These lots are then being awarded to the bidder with the highest bid.

However, sellers on eBay are currently only able to enhance an auction with a Buyout Option when offering a single good. The only ways to offer multiple goods are thus either by a standard multi unit uniform price auction or by a fixed price listing, and thereby not enabling the seller to use a hybris of selling mechanisms when tendering multiple goods.

**Amazon**

Other than offering goods on its traditional fixed price online platform, sellers can choose to sell their items by way of an open ascending price auction at Amazon.[20] The Buyout Option by which bidders can call an early halt to the auction is called "Take-It Price" at Amazon. Sellers can alter the price of the option so long as no bid has been submitted. The option cannot be changed thereafter and remains available until either a bidder submits a bid equal to the Take-It Price or the auction ends and the good is awarded to the bidder with the highest bid. The Buyout Option available on Amazon can therefore be classified as a permanent Buyout Option. Furthermore, bidders can use "Bid-Click", a proxy bidding tool offered at auctions. At Amazon, even though the auction duration must be ex ante specified by the seller, auctions are enhanced by the so called "Going, Going, Gone" feature that automatically extends an auction by another 10 minutes from the time of the latest bid submitted whenever an appropriate bid is cast in the last 10 minutes before the closing time of an auction. Thereby, they offer bidders to challenge late bids which can be beneficial for the seller since the risk of potential high bids towards the end of an auction being ruled out by an early termination of the process is reduced and thereby abating the potential detriments of bid sniping.

In what is again called a "Dutch Auction", sellers have the possibility to simultaneously list multiple identical items for sale. Similar to the multiple item auctions at eBay, bidders at Amazon can submit bids specifying both the quantity requested and

[20] See www.amazon.com.

the price they are willing to pay. Winning bidders likewise pay the lowest winning bid and the allocation of the goods follows the level of bids successfully submitted in decreasing order. Bid rationing is again possible if demand exceeds supply at the final auction price. However, unlike in the eBay auction, bidders are bound to purchase any number of goods they successfully bid for despite the fact that they may receive fewer units than requested. As it is the case at eBay, sellers at Amazon currently cannot enhance multi unit auctions with a Buyout Option.

### Tazbar

Tazbar offers four different types of auctions.[21] These are: (i) Standard auctions, (ii) instant purchase auctions, (iii) multiple item instant purchase auctions and (iv) combinatorial standard and instant purchase auctions. In a *standard auction*, bidders can submit traditional bids over a period of time ex ante set by the seller with the bidder submitting the highest bid winning the auction. The "book bids" feature allows bidders to use an agent that is de facto a proxy bidding tool, similar to the ones offered at eBay and Amazon. Furthermore, analogous to the auctions on Amazon, an auction is automatically extended by 30 seconds if a successful bid has been submitted in the last 30 seconds of the prespecified duration of the auction to help preventing bid sniping. In *instant purchase auctions*, sellers can list goods for a predetermined period of time at a fixed price. Bidders can then choose to agree to pay the given price and receive the good instantly and thereby ending the auction. This type of auction could therefore rather be classified as a conventional fixed price offering. The *multiple item instant purchase auction* allows sellers to offer multiple identical goods over a specific period of time, but at a fixed price. The multiple item instant purchase auction ends when either the bidding time is over or all goods have been sold. Thus, the notation of this type of auction is rather miserable and misleading since it effectively is a simple fixed price offer and not an auction in the traditional sense. What is referred to as *combinatorial standard and instant purchase auctions* is actually a combination of both the standard auction and the instant purchase auction. In this hybris of a fixed price offering and a traditional auction, the seller can offer a Buyout Option denoted "instant purchase price" that can be exercised by any bidder and thereby terminating the auction. The Buyout Option disappears as soon as a bid above the reserve price has been submitted, thereby making it a temporary Buyout Option. These instant purchase prices can however not be used at the simultaneous sale of multiple items.

[21] See www.tazbar.com.

Tazbar furthermore offers another type of auction, called *wanted auctions*. In a wanted auction, a potential buyer can place an advert for a good he wishes to acquire.[22] Sellers can subsequently place offers against that auction where the seller with the lowest price wins. The auction ends as soon as either the offer period reaches an end or the buyer chooses to accept an offer submitted. If the good is not purchased by the buyer and remains unsold, it is being automatically moved to the instant purchase auction section of Tazbar's platform. Other potential buyers, while not being able to place bids for the same good during the auction, can submit sealed bids that come into effect if the good is moved to the instant purchase auction, with the highest bidder winning the good. Thereby, a system is created where, in contrast to most other auction sites, the inclusion of the demand side as auctioneers is captured. Furthermore, this type of auction can indeed be classified as a Dutch auction in the traditional sense since the price starts at a high level that is subsequently being lowered until it is eventually accepted (by the potential buyer in this specific case).

**Yahoo!**

The auction mechanism available on Yahoo! platforms is briefly being presented despite the fact that the US and Canada auction sites were closed down on June 16, 2007.[23] Sellers at Yahoo! can as well enhance their auction by permanent Buyout Options, called "Buy Prices". Auctions can further be endowed with an extension feature that allows auctions to automatically continue so long as bidding is active. On the other hand, sellers can choose what is called a "hard close time" that defines a specific ending time of an auction. Yahoo! does offer an automated proxy bidding system that can be used by bidders to let the agent bid for them. Moreover, it is stated on Yahoo!'s auction pages that sellers can choose to simultaneously offer multiple identical items in a single auction. However, on all three remaining sites still in operation, not a single multi unit auction could be observed and no further details were disclosed on the multi unit auction format.

[22] These auctions are also called "Reverse Auctions" in auction theory. See Milgrom (1987a) and Wang (1993).

[23] See www.yahoo.com. The closing of these auctions sites had so far no impact on the Yahoo! auction sites in Hong Kong, Singapore and Taiwan. Yahoo! especially deserves to be mentioned since several authors have used its permanent Buyout Option to compare it with temporary Buyout Options. See Lee and Ahn (2004), Reynolds and Wooders (2005) and Gupta and Gallien (2007).

**Ricardo**

Ricardo offers a seller to either tender his goods by a fixed price offer, by a standard ascending price auction or by an auction enhanced by a permanent Buyout Option.[24] Obviously, if an auction is enhanced by a Buyout Option, bidders can terminate the auction by exercising the option and receiving the good instantaneously. This platform allows an auction to extend its prespecified duration by 5 minutes if a successful bid has been submitted within the last 5 minutes of the auction. In the multi item auction format available at Ricardo, sellers can offer multiple identical goods for which bidders can submit bids that specify the quantity as well as the price they are willing to pay for each good. The goods are then awarded to the bidders submitting the highest bids. The final price for the goods is again determined by the lowest winning bid and identical for all winning bidders. If several identical bids have been submitted, the bidders submitting successful bids earlier receive the goods whilst bid rationing is possible when demand exceeds supply at the final price. In contrast to other auction sites described earlier, proxy bidding can be used in both single unit and multi unit auctions. The Buyout Option in turn can only be added in single unit auctions.

**Overstock**

At Overstock, sellers can enhance their auction by the "Make It Mine" feature that is available to bidders so long as no bid has exceeded its stated reserve price, thus a temporary Buyout Option in the traditional sense.[25] If the Buyout Option is not being exercised by any of the bidders, the good is sold by way of a standard ascending price auction with proxy bidding. An auction at Overstock is also automatically being extended for an additional 10 minutes if a successful bid is submitted within 10 minutes of the auction closing time. Overstock currently does not offer any kind of multi item auctions where multiple goods can be simultaneously put for sale.

**uBid**

Sellers at uBid can choose between offering goods by way of a traditional ascending price auction, an auction enhanced by a permanent Buyout Option or by simply posting a fixed price without the possibility of competitive bidding.[26] Furthermore,

[24] See www.ricardo.ch.
[25] See www.overstock.com. The Buyout Price at Overstock must at least be 1.5 times the auction's reserve price.
[26] See www.ubid.com.

they can simultaneously offer an arbitrary amount of identical items in one single auction. When simultaneously offering multiple items in an auction, the winning bids are determined according to the bid prices successfully submitted, the quantity requested (where larger quantities are given precedence) and the initial bid time (where earlier bids win).[27] Successful bidders at uBid however do not pay a price determined by the lowest winning bidder as on the other auction sites listed here, but instead have to pay their individual price submitted in the auction. Therefore, the multi unit auction format uses a discriminatory pricing rule. The duration of uBid auctions are moreover being indefinitely extended by 10 minutes if a successful bid is submitted within the last 10 minutes before the end of an auction. The "Bid up to my Maximum Bid" or "Bid Butler" feature offered at most auctions allows the bidders to submit bids by means of a proxy bidding tool instead of placing a single bid. Bidders in the auction can buy the items immediately by executing the "uBuy It" feature, even when multiple items are being offered. Thus, it is the only of the auction sites mentioned in this chapter that actually allows for the sale of only part of the total supply by a permanent Buyout Option in a multi unit auction and at the same time selling the remaining items by a traditional auction.

**eBid**

In contrast to uBid, the Buyout Option offered at eBid, denoted "BuyNow", is only available during the opening period of an auction before any bidding has effectively started, thus making it a temporary Buyout Option. However, the Buyout Option can also be used for multi unit auctions, where it is likewise only available before the first competitive bid is being submitted. In the multi unit auction format, all successful bidders pay the same price, which is equal to the lowest winning bid. Proxy bidding, even though available, can only be used at single unit auctions and does not exist when several goods are simultaneously at sale. Similar to Tazbar, eBid further allows bidders to post a "Wanted Auction", stating their willingness to buy a certain good for at most the posted price where the winning seller is the one submitting the lowest price.

The different auction formats and use of Buyout Options on the auction platforms presented beforehand are summarized in Table 3.2. As can be seen from Table 3.2, out of the auction sites listed above, only uBid and eBid offer Buyout Options for the simultaneous sale of multiple goods while uBid is the only platform that uses a discriminatory mechanism to determine prices at its multi unit auction alternative.

[27] If the total quantity of goods requested by successful bidders exceeds the number of units offered, certain bidders might receive only a partial quantity of their demand.

Table 3.2: Use of Buyout Options in online auctions

| | Pricing Rule | | Buyout Option | | Buyout Option |
|---|---|---|---|---|---|
| Auction Site | Uniform | Discr. | Temporary | Permanent | for Multiple Units |
| Amazon | x | | | x | |
| eBay | x | | x | | |
| eBid | x | | x | | x |
| Overstock | | | x | | |
| Ricardo | x | | | x | |
| uBid | | x | | x | x |
| Yahoo! | ? | | | x | ? |
| Tazbar | x | | x | | |

Despite the fact that Buyout Options have so far only widely been offered on online based auction platforms, their potential range of application is in principle unlimited. Wherever an auction is in place to market any kind of services or goods, Buyout Options could be a valuable device to optimize the outcome of transactions, ranging from business-to-business procurement auctions to business-to-consumer marketplaces providing forward auction services and from financial market auctions to consumer-to-consumer auctions such as auctions of art. Time will tell whether Buyout Options will experience a broader application on real-world auction markets.

# Chapter 4

# Related Literature on Buyout Options in Auctions

While literature on auctions is large, existing research on Buyout Options in auctions is relatively novel and limited. The theoretical, empirical and experimental work on Buyout Options and its key findings completed to date is descriptively outlined hereafter to provide a sound overview of the fundamentals the subsequent model in chapter 5 is based on.

Lucking-Reiley (2000) was the first to observe the use of Buyout Options in auctions in an exhaustive survey of internet auctions that were being used as of 1999. Further to explicitly mentioning the analysis of the consequences of enhancing an auction by a Buyout Option, Lucking-Reiley quotes evidence that indeed the execution of these options in online auctions is relatively frequent.

## 4.1 Theoretical Literature

At the outset of the theoretical literature on Buyout Options in auctions are considerations of auction participants' risk attitudes towards the acquisition and sale of the good at stake. Recall that risk aversion on either side of the transaction is one of the two intuitively apparent factors that may reason the enhancement of an auction with a Buyout Option as well as its execution.

Budish and Takeyama (2001) consider an independent private values ascending price auction for a single good where a revenue-maximizing seller faces two bidders. The bidders may either have a high valuation or a low valuation for the good at sale with exogenously given probabilities. While the seller does not post a reserve price, he can enhance the auction by a permanent Buyout Option. They show that in this setup a seller would only receive higher expected revenue if bidders are risk averse while when bidders are risk neutral, the expected revenues are equivalent in the auctions with and without a Buyout Option. They further find that with risk averse bidders, the seller's expected revenue in the ascending price auction with a properly set Buyout Price can exceed that from a standard first price sealed bid auction. Thus, given bidder risk aversion, the Revenue Equivalence Theorem does no longer hold, even for the case where no Buyout Option is being offered.[1] The key argument for this outcome is that by reducing risk for some high value bidders, the seller can extract a premium by offering such a Buyout Option.

Reynolds and Wooders (2005) extend the results presented by Budish and Takeyama (2001) by presenting a more general model for a single unit ascending price auction with a reserve price and a Buyout Option where bidder valuations are independently and identically drawn from a continuous distribution. They characterize equilibrium bidding strategies for both temporary and permanent Buyout Options. The analysis shows that if bidders have constant absolute risk aversion, both the auction enhanced by a temporary Buyout Option and the auction with a permanent Buyout Option are payoff equivalent from the bidders' perspective. Bidders having decreasing absolute risk aversion however prefer the auction with a temporary Buyout Option while bidders with increasing absolute risk aversion prefer the auction with a permanent Buyout Option. They find that if bidders are risk averse, enhancing the auction with a Buyout Option raises expected seller revenue for a wide range of Buyout Prices as the bidders are willing to pay a price including a risk premium rather than bearing the risk of not winning the auction or winning the auction at an uncertain price. If bidders are instead risk neutral, ceteris paribus, both the auction with a temporary and a permanent Buyout Option yield the same expected seller revenue, that is, assuming identical reserve prices and Buyout Prices. If bidders have constant or decreasing absolute risk aversion, then the auction with a permanent Buyout Option raises more seller revenue than the auction with a temporary Buyout Option. Additionally, expected seller revenue is the same in the ascending price auction without a Buyout Option as in the auctions enhanced by a Buyout Option if the Buyout Price is set at a level such that it is not being exercised at the start of the auction, given the same reserve price.

A model of a single unit second price sealed bid auction with a reserve price enhanced by a permanent Buyout Option is studied by Hidvégi et al. (2006). Bidders again have independent private valuations randomly drawn from a common cumulative

[1] See also Vickrey (1961), Vickrey (1962), Reiley and Samuelson (1981) and Myerson (1981).

distribution function. The strategy space for all bidders with a valuation above the Buyout Price is characterized by four possible strategies, that is: (1) To traditionally bid up to their valuation, (2) to keep bidding until winning the auction or an individual bidder's threshold Buyout Price is reached and then immediately exercise the Buyout Option, (3) to conditionally bid their valuation but immediately exercise the Buyout Option if at least one other bidder submits a bid or (4) to unconditionally exercise the Buyout Option. It is being shown that there exists an equilibrium with unique reference points where bidders with a valuation above the Buyout Price and below the lower of the two reference points exercise the Buyout Option as soon as the highest bid reaches a threshold price, bidders with valuations between the reference points exercise the Buyout Option conditional on the existence of a valid bid at least as high as the reserve price and bidders with valuations above the upper reference point unconditionally exercise the Buyout Option. The bidders' equilibrium threshold Buyout Prices are strictly decreasing in their valuation so that the bidder with the highest valuation for the good will reach his threshold value first and thereby guaranteeing an efficient outcome of the auction since such a bidder will first exercise the Buyout Option. Furthermore, the more risk averse a bidder, the lower his threshold Buyout Price, thus implying that the seller can gain higher expected profits since these bidders tend to exercise the Buyout Option earlier. Hidvégi et al. (2006) show that risk averse bidders are not necessarily better off in such an auction with a Buyout Option since they try to reduce the risk of losing the auction by bidding for the Buyout Price and therefore potentially paying more than they would in a standard second price sealed bid auction without a Buyout Option. They prove that when bidders are risk neutral or uniformly risk averse, their expected utility in a second price sealed bid auction enhanced with a Buyout Option is the same as in the standard auction without a Buyout Option if the Buyout Price is properly set above the lower bound. In contrast, the seller's revenue is higher in the auction with a Buyout Price when bidders are risk averse. Moreover, if the seller is risk averse, he always prefers the auction with a Buyout Option since then his expected utility is never lower than in the standard auction setting without a Buyout Option.

Most relevant for the subsequent analysis of a simultaneous multi unit auction enhanced by a Buyout Option is the Model introduced by Mathews and Katzman (2006). Mathews and Katzman (2006) focus on the impact of risk aversion on the part of the seller in a single unit auction with a temporary Buyout Option. In their model, a seller offers a good to risk neutral bidders in a second price sealed bid auction enhanced by a temporary Buyout Option. A risk averse seller, regardless of his degree of risk aversion, will always prefer being able to offer such a Buyout

Option compared to the standard auction format. To maximize his expected utility, a risk averse seller offers a Buyout Option low enough so that it is being exercised with positive probability. The auction may result in an ex ante pareto improvement if all bidders are better off when a Buyout Option is being offered. However, with positive probability there exist bidders with relatively high valuations that are worse off when the seller is able to offer a temporary Buyout Option and the allocation of the good may be inefficient.

Chen et al. (2006) propose a dynamic single unit ascending price auction with a permanent Buyout Option where bidders' valuations are independently drawn from a uniform distribution. Their study focuses on the effects of sellers' and bidders' risk attitudes towards the execution of the Buyout Option. They find that if both the seller and the bidders are risk neutral, the seller receives the same expected revenue when offering or not offering a Buyout Option. Thus in this case, the two auctions are revenue equivalent. However, when either the seller or the bidders are risk averse, the seller's expected utility is strictly higher when offering an appropriately set Buyout Price since it reduces the risk that both the seller and bidders face in the bidding process. Bidders on the other hand only have a higher expected utility if they have very high valuations despite the fact that the Buyout Option might be beneficial for them by offering them an option to immediately buy the good at an ex ante determined price. Chen et al. (2006) additionally argue that the offering of a Buyout Option increases competition amongst bidders. They further find that the optimal equilibrium Buyout Price is increasing in the bidders' degree of risk aversion and at the same time decreasing in the seller's degree of risk aversion. Empirical data from 2'182 observations collected from Taiwan's Yahoo! auctions furthermore finds evidence for three testable implications of the theoretical model, namely that, on average, (i) the transaction price of homogeneous goods is higher when an auction is enhanced by a Buyout Option, (ii) the transaction price of a good is increasing in its Buyout Price and (iii) the longer it takes for a good to be sold, the lower will be its transaction price.

Klumpp and Ranger (2006) consider a single unit ascending price auction with independent private bidder valuations enhanced with a permanent Buyout Option but without a reserve price. They extend previous research by analyzing the auction mechanism in a more exhaustive way, taking into account that in such auctions bidders usually have more information available on which to condition their bidding behaviour. In particular, they argue that in most ascending auction formats bidders continuously observe how many of their competitors and at what price they drop out of the auction. Therefore, in equilibrium, bidders revise their threshold Buyout Prices every time a bidder drops out of the auction making their behaviour dependent on the history of the auction and thereby resulting in a lower likelihood of bidders exercising the Buyout Option immediately at the start of the auction. Thus, bidders tend to bid less aggressively in the model proposed by Klumpp and Ranger (2006). Nevertheless, they show that with risk averse bidders, it is possible for the seller to set a Buyout Price lower than the highest bidder's valuation for which his ex ante expected revenue is higher than in the standard auction format without a Buyout Option.

In a simple model of an ascending price auction with independent private bidder valuations, Jung and Kim (2004) show that a seller can strictly increase his expected revenue when offering a Buyout Option even in the case of risk neutral bidders. They show that if bidders' valuations are drawn from a uniform distribution, the revenue gain from employing a Buyout Option decreases in the number of potential bidders participating in the auction. However, the model proposed by Jung and Kim (2004) is characterized by strong limitations that make their results less significant, even from a theoretical point of view.

A further yet decisive alternative explanation for the existence of Buyout Options in auctions is time impatience on either side of the transaction. If auction participants discount the value of future transactions, it may be optimal to introduce a Buyout Option since, on one hand, the seller may be willing to pay a premium to sell the good early or, on the other hand, the bidders may be willing to pay a premium to receive the good quickly.

Mathews (2004) analyzes a single unit ascending price auction with a temporary Buyout Option that allows for the study of the impact of time discounting on both sides of the transaction. Again, bidders' valuations are independently drawn from a common continuous uniform distribution function and are private information. Mathews (2004) finds a symmetric equilibrium in which the seller sets a Buyout Price high enough so that no bidder will exercise the Buyout Option if neither the seller nor the bidders do in fact discount future transactions. However, if the seller is time impatient, he will choose a Buyout Price low enough so that the Buyout Option is exercised with positive probability. Similarly, if potential bidders are time impatient, the seller maximizes his expected revenue by choosing a Buyout Price low enough for the Buyout Option to be exercised with positive probability, regardless of whether the seller discounts future transactions or simply maximizes expected revenue. Thus, by offering an appropriately set Buyout Price, the seller can strictly increase his expected utility if time impatience exists on either side of the transaction. Finally, allowing a time impatient seller facing bidders that are not time impatient to offer a Buyout Option results in an ex post improvement compared to the auction where the seller is not able to offer a Buyout Option.

Mathews (2006) presents a more generalized version of the model developed in Mathews (2004) by relaxing the assumption of uniformly distributed bidder valuations in order to examine in more detail the ex ante welfare implications of auction participants when a temporary Buyout Option is in place. It is shown that if the seller is either risk averse or time impatient, allowing him to offer a Buyout Option will increase his utility. A risk averse seller will always choose a Buyout Price low enough so that it is being exercised with positive probability. Despite the fact that the enhancement of the auction by a Buyout Option may reduce expected revenue, a risk averse seller will find the altered mechanism beneficial since it reduces the risk induced by selling the good at a lower price in a standard ascending price auction without a Buyout Option. A seller discounting future transactions will as well choose a Buyout Price such that the Buyout Option is exercised with positive probability. Again, the Buyout Option decreases his expected revenue but potentially allows the transaction to occur sooner and thereby increasing his utility. If however the seller is neither risk averse nor time impatient, he will not find it beneficial to offer a Buyout Option. On the other hand, when examining bidder welfare, the impact of a temporary Buyout Option critically depends on the distribution from which their valuations are drawn. If the auction results in an ex ante pareto improvement, clearly all bidders are better off when the seller offers a Buyout Option. It is however possible that, again, bidders with relatively high valuations are worse off if the seller offers such an option. Compared to the standard ascending price auction mechanism, the enhancement of the auction by a Buyout Option is only beneficial whenever the probability density function is non-decreasing in bidder valuations. In fact, then an auction with a Buyout Option results in an ex ante pareto improvement compared to an auction without a Buyout Option.

Gupta and Gallien (2007) compare temporary and permanent Buyout Options using a unified modeling framework capturing the impact of the auction participants' time sensitivity, where the utility of both the seller and bidders are being discounted by a respective time discounting factor. They consider a monopolistic seller offering a single good in a second price auction with a time limited bidding period facing an arrival stream of potential bidders which is not observable per se but follows a Poisson process with a well known exogenously given constant entry rate. Bidders' valuations are assumed to be independently drawn from a distribution with compact support. Furthermore, all auction participants are assumed to be risk neutral. The auction mechanism does not contain a concealed reserve price but has a minimum required bid that is being defined by the lower bound of the distribution of bidders' valuations which thus effectively corresponds to a publicly posted reserve price since no bids below that value are being accepted. It is additionally assumed that every bidder knows the payment that would be made by the winning bidder if the auction would be immediately terminated subsequent to his arrival at the auction. Every bidder has the possibility to exercise the Buyout Option at any given time between his arrival and the end of the auction, given that the Buyout Option is still available. The analysis and the numerical experiments presented by Gupta and Gallien (2007) reveal that the optimal Buyout Price for the seller increases in the expected number of bidders and in the bidders' time sensitivity while at the same time decreasing in the seller's time sensitivity. They also suggest that in the market environment in which their model is embedded, the value of the permanent Buyout Price maximizing expected seller utility exceeds the Buyout Price he would choose when offering a temporary Buyout Option. They further find that a time sensitive seller facing only a few patient bidders should exclusively use a fixed price offer instead while a patient seller facing many bidders should bypass the Buyout Option and only use a regular auction mechanism. For market environments in-between those two extremes, the hybrid use of an auction and a fixed price offer in the form of a Buyout Option is appropriate. The equilibrium analysis by Gupta and Gallien (2007) suggests that when the seller offers a temporary Buyout Option, the first bidder to submit a regular bid and not exercising the Buyout Option will do so immediately upon his arrival. However, when offering a permanent Buyout Option, bidders will only submit regular bids shortly before the end of the auction.[2] Therefore, the marginal impact of a permanent Buyout Option relative to a temporary Buyout Option is negative for the seller if bidding activity in the auction attracts more bidders, whilst the relative attractiveness to offer a temporary Buyout Option decreases with the expected number of bidders while it is increasing in the case of a permanent Buyout Option. They further find that the seller's expected revenue from the auction with an optimally set permanent Buyout Price exceeds the expected revenue obtained in the auction with an optimal temporary Buyout Option.

In addition to risk aversion and time impatience, other significant transaction costs may be associated with the participation in auctions. The impact of substantial transaction costs that may emerge when participating in an auction can thus further

[2] See bid sniping in section 2.7 of this thesis.

explain the use and potential benefits of Buyout Options in auctions.

Wang et al. (2004) examine a single unit second price sealed bid auction with a reserve price and a temporary Buyout Option with a risk neutral seller. The bidders whose private valuations are independently drawn from a uniform distribution have participation costs that only incur upon submitting a feasible bid. They find that an increase in bidders' participation costs can reduce the seller's expected revenue when offering the good without a Buyout Option. However, when enhancing the auction with a Buyout Option, both the seller's expected revenue and bidder utility can be increased versus an auction where no Buyout Option is available if bidders' participation costs are substantial since the Buyout Option reduces these costs by making a participation profitable even for bidders who might not have been able to gain positive payoffs in a setting without a Buyout Option.

A further argument justifying the use of Buyout Options in auctions is the idea that sellers may use them to intertemporally optimize their revenues given that homogeneous goods are sequentially being sold at different points in time or on different markets. As it can be observed from many online auction sites, identical goods are being sold at different times and in different auctions, thus reasoning the consideration of intertemporal optimization and alternative competing markets.

In an early contribution, Lopomo (1998) studies simple sequential auctions. Even if Buyout Options are not explicitly being mentioned, the mechanism examined can be regarded as an auction with a permanent Buyout Option that potentially changes its price (however it cannot increase). The seller facing risk neutral bidders is limited to choose a sequence of increasing bid prices and non-increasing Buyout Options. It is being shown that in the proposed setup, the seller will optimally choose a sequence of Buyout Prices (or ask prices, in the terminology of the author) that are never being exercised by any of the bidders to maximize his expected revenue.

Kirkegaard and Overgaard (2007) examine a dynamic environment of ascending price auctions where multiple sellers compete to sell a single good to bidders with multi unit demand. Their analysis thereby broadens the view to the extent that bidders and sellers alike are aware of the fact that similar products or close substitutes might be offered in the future as it can be observed in several of today's online auction markets. They examine a model in which two homogeneous goods are sequentially being offered for sale and where sellers face bidders with independent private valuations featuring decreasing marginal utility. A temporary Buyout Option can only be offered by the first seller and in both auctions reserve prices are being ignored. If no seller chooses to offer a Buyout Option, the expected revenue from the second auction is higher than in the first auction. Both auctions follow an ascending price mechanism in which the prices are determined by the respective highest losing bids, in their model thus by the bidders with the second highest marginal valuations. Since bidders feature multi unit demand, the rival bidders' marginal valuations are relevant in that they lead to increasing expected revenues due to the fact that the second highest bidder underestimates the intensity of competition and consequently the bid necessary to win the second auction. If however the first seller chooses to offer a Buyout Option, he clearly has an incentive to extract revenue from the subsequent auction by appropriately setting the Buyout Price. It is being shown that for such a seller it is optimal to set a Buyout Price that is being exercised with positive probability leading to an increase in his expected revenue versus the revenue he could have gained in the standard setting without a Buyout Option. Kirkegaard and Overgaard (2007) find that the seller offering the first good can always increase his revenue by introducing an appropriately set Buyout Price whereas the second seller's revenue is adversely affected as is overall revenue from both auctions combined. Furthermore, the availability of a Buyout Option in the first auction in their model introduces ex post inefficiency in that bidders who would not have won a good in the standard setting can indeed win a good with positive probability.

Lee and Ahn (2004) were the first to analyze ascending price auctions with Buyout Options in the presence of competing fixed price markets. In their approach, bidding in the auction or exercising the Buyout Option is not the only way to get the good but potential bidders have the possibility to buy identical goods on alternative fixed price markets, shedding light not only on whether to bid in the auction or exercise the Buyout Option, but also on where to buy the good. Thereby, they introduce an outside option for all potential bidders that needs to be accounted for in the analysis of optimal bidder and seller behaviour. Lee and Ahn (2004) examine an ascending price auction where a risk neutral seller offers a single good to multiple risk neutral bidders with independent private valuations drawn from a standard uniform distribution. When offering a temporary Buyout Option, they find that the seller would optimally set the Buyout Price below the lowest available market price since otherwise the Buyout Option would not have any effect as bidders would instead prefer buying at their outside option. Bidders will thus only exercise the Buyout Option if it is set low enough to compensate them for the benefit the seller could expect from the bidders being engaged in participating in the auction by actively bidding. On the other hand, they find that when the seller offers a permanent

Buyout Option, bidders with a valuation above a certain threshold value given by the equilibrium condition can experience a utility loss since they are compelled to bear more risk of being outbid by bidders with a lower valuation exercising the Buyout Option. With regard to expected seller revenue, when offering either a permanent or a temporary Buyout Option, a seller can at most gain a revenue equal to the auction without a Buyout Option when appropriately setting the Buyout Price. Therefore, enhancing the auction by a Buyout Option does not lead to an increase of expected seller revenue.

Bose and Daripa (2006) focus on the role of auctions for sellers who use these in addition to traditional fixed price sales to be able to expand their market base to potential buyers with lower valuations that might not want to buy a good at the fixed price. They examine a scenario in which bidders with valuations above a specific threshold value drawn from a cumulative distribution function buy the good through the posted price mechanism and buyers below the threshold valuation compete in an auction with a temporary Buyout Option. A model with a risk neutral seller and two ex ante symmetric risk neutral buyers with private valuations is analyzed. Bose and Daripa (2006) find that the optimal mechanism involves a posted price selling followed by a second price sealed bid auction enhanced by a temporary Buyout Option with pooling at the top of the distribution of bidders' valuations.

## 4.2 Empirical and Experimental Literature

Despite the increasing number of theoretical considerations of the use of Buyout Options in auctions, empirical literature has to date been relatively sparse.

Durham et al. (2004a) conduct a field experiment combined with data from eBay auctions for 2001 American Eagle Silver Dollars to examine the use and execution of temporary Buyout Options. They study the impact of bidder and seller reputation and the level of the Buyout Price on bidder behaviour. They find that sellers who choose to offer the goods in an auction with a Buyout Option are typically sellers with a relatively high reputation, assuming that the reputation score is a proxy for seller experience in the market and familiarity with the auction procedure, since it is fairly well correlated with the number of transactions completed antecedent to the auctions observed.[3] Furthermore, the probability of a Buyout Option to be exercised increases in seller reputation and decreases in the level of the posted Buyout

[3] Sellers and bidders on eBay can post comments and rankings on members they have bought from or sold to after the conclusion of an auction. This information, along with an overall feedback score, is then being disclosed to all members and can be regarded as an individual member's reputation.

Price. They also find that a seller without any reputation considerably changes bidder behaviour in that his reputation lowers the average selling price, regardless of the presence of a Buyout Option. A reputation of zero does as well reduce the likelihood of bidders to accept the Buyout Price. Moreover, bidder reputation is significant in auctions with a Buyout Option as bidders are more likely to exercise the Buyout Option with increasing reputation likely being a result of them having more experience and recognizing that the Buyout Option may reduce risk on their side. Finally, the data suggest that the average price effectively paid in the auctions is positively related to seller reputation while at the same time negatively related to bidder reputation. Additionally, the data show that, on average, the use of Buyout Options does indeed tend to raise seller revenue. In a subsequent study, Durham et al. (2004b) show that bidders do not always take the advantage of executing a Buyout Option even when the Buyout Price is set below the prevailing market price.

Anderson et al. (2004) analyze data gathered from the sale of Palm Vx handheld computers on eBay. When comparing all auctions, the probability of the sale of a good is somewhat more likely when sellers offer a Buyout Option. The data reveal that when sellers offer new or undamaged goods, the Buyout Prices are, on average, higher versus the case when damaged goods are being offered. Moreover, they find that auctions where larger quantities of the good were available have lower average Buyout Prices and are also less likely to be determined by the execution of the Buyout Option. With regard to the entire data set, Anderson et al. (2004) show that the existence of a Buyout Option does not have a significant effect on the final sales price. However, when examining the subset of auctions that were enhanced by a Buyout Option, when the Buyout Price was accepted by a bidder, the goods were on average sold at higher prices and thus leading the sellers offering the option to obtain higher revenues.

Hendricks et al. (2005) use data collected from Texas Instruments TI-83 calculators auctioned on eBay of which nearly one third offered a Buyout Option. They find that the auctions where a Buyout Option was offered on average generated more revenue than the auctions where such an option was not in place. However, since the auctions with a Buyout Option on average had significantly higher reserve prices, it is not clear whether the Buyout Option alone was accountable for the higher average revenue.

Ackerberg et al. (2006) develop a structural model for bidding in auctions with a temporary Buyout Option and apply it to data collected from the sale of portable computers on eBay. The data indicate that the average sales price does not significantly differ in the auctions where a Buyout Option was being offered and where not. However, seller revenue is on average higher in auctions where the Buyout Option was indeed exercised. They further note that the information on the good provided by the seller is less in the auctions without Buyout Options and that the average length of the auctions with a Buyout Option is higher than in the standard setting where sellers do not offer such an option. In addition, with regard to the auctions enhanced by a Buyout Option, the proportion of auctions where bidders exercised the Buyout Option is higher for auctions where sellers choose a higher length of the ex ante determined duration of bidding time.

Chan et al. (2006) study the effect of Buyout Prices on bidders' willingness to pay in auctions with a permanent Buyout Option. From the data of 2'322 notebook auctions conducted on one of the largest auction sites in Korea, they find that by setting a Buyout Price higher than the expected price of the good, a seller can increase bidders' willingness to pay whereas by lowering the Buyout Price he reduces it. In their empirical analysis they show that a significant number of sellers set Buyout Prices that do not maximize their revenue by either setting the Buyout Price too high or too low and claim that this suboptimality reflects the sellers' misinterpretation of competition amongst the goods that are concurrently being offered at the different auctions. They further claim to offer a model that allows for the derivation of an optimally set Buyout Price.

Song and Baker (2007) contribute to the empirical literature by introducing a taxonomy of variables exclusively controlled by the seller rather than variables under the control of bidders. Amongst others they test if the presence of a Buyout Option is associated with lower seller revenue and, more generally, if the presence of a Buyout Option will negatively influence the association between the number of bids and seller revenue. They further test whether the availability of Buyout Options has a positive effect on the association between the initial bid price and seller revenue. By using data on 378 DVD movie auctions and 412 MP3 player auctions on eBay, they find that the Buyout Price is not a significant predictor for seller revenue since it does not provide additional value or information for bidders competing for goods that are widely available and whose costs can be easily deducted from existing fixed price markets (such as for DVD movies and MP3 players). However, the negative moderating effects of a Buyout Option on the relationship of the number of bids and the final closing price of the auction as well as the positive effect of the option on the relationship between the initial bid price and the closing price are found to be highly significant.

The impact of Buyout Options in online auctions with independent bidder valuations on efficiency, participant revenues and bidding behaviour is investigated by Peeters et al. (2007). By conducting a laboratory experiment, they find that allocative efficiency is unambiguously reduced when enhancing an auction with a temporary Buyout Option. In contrast to a wide range of theoretical models, they show that Buyout Options can reduce seller revenue in that it reduces the final price of the auction when the Buyout Option is not exercised in an environment where bidders only know the distribution of their valuation. Their main argument is that when bidders are uncertain about their own valuation, the Buyout Option is utilized as an anchor for bidding behaviour in the auction. However, when bidders know their private valuation, an overall price reduction is not a robust result and is not confirmed by the data. They further show that bidders in the experiment were unwilling to post bids exceeding the Buyout Price.

Grebe et al. (2006) conduct an experiment using an unchanged real world market environment by inviting participants who already used the eBay platform to study seller and bidder behaviour in ascending price auctions with a temporary Buyout Option. They find that even experienced sellers choose Buyout Prices lower than they would optimally set to maximize revenue as well as bidders systematically underbidding their true valuation. The reason for sellers tending to lower their Buyout Prices is to be found in their experience, that is, after unsuccessfully offering goods in previous auctions, they reduce the Buyout Price which results in foregoing potential revenue.

The properties of temporary Buyout Options in private and common bidder valuation auctions are experimentally tested by Shariar and Wooders (2006). For the case of pure private bidder valuations, they find that the use of a Buyout Option has a positive and statistically significant effect on seller revenue along with a reduction of the variance of seller revenue, thus being beneficial for the seller consistent with theoretical predictions if either the seller or bidders are risk averse. On the other hand, in the case of common values, their results show that Buyout Prices have a small positive though statistically insignificant effect on seller revenue and its variance.

The survey of prevailing literature on Buyout Options in auctions given in this chapter has highlighted the numerous facets that can be addressed using theoretical, empirical as well as experimental methods. Existing literature has so far primarily examined the effects of risk aversion, time impatience and participation costs of auction participants and the consideration of alternative competing markets on the outcome of auctions enhanced with Buyout Options. The contemplated considerations may justify the implementation of Buyout Options in auctions since they may be beneficial for all participants leading to a potential improvement of auction results. At the same time however, literature suggests that despite its obvious beneficial implications, the existence of Buyout Options may give rise to the possibility of inefficient auction outcomes since the goods may be allocated to bidders that do not value them most when the options are indeed being exercised.

It further points to the fact that the topic of Buyout Options in auctions is a subject that has not only recently attracted attention but that there still is considerable research required to contribute to a sound comprehension of the impact such options have on the outcome of alternative auction formats.

Moreover, the survey on existing literature indicates that single unit ascending price and sequential single unit auctions enhanced with a Buyout Option have hitherto predominantly and extensively been studied. Auctions in which identical goods are simultaneously being offered have so far not been discussed despite the fact that such auctions do in fact exist in practice. The aim of the subsequent chapters is to extend existing work on Buyout Options in auctions to the case for the simultaneous sale of multiple goods. A multi unit Vickrey auction with discriminatory pricing enhanced with a temporary Buyout Option where a risk averse seller simultaneously offers two identical goods to risk neutral bidders with single unit demand will be examined, thereby making it a hopefully valuable contribution to existing literature on Buyout Options in auctions.

# Chapter 5

# The Model

## 5.1 Model Setup

In this section an analytical model is developed to study the optimal equilibrium strategies of the seller and the bidders in an auction enhanced with a temporary Buyout Option, where two homogeneous goods are simultaneously being offered for sale. The following analysis is based on the model introduced by Mathews and Katzman (2006).

There are $n+1$ ex ante symmetric risk neutral bidders (with $n \geq 3$) with single unit demand competing in a multi unit Vickrey auction with discriminatory pricing.[1] Each bidder has a private valuation for the good denoted $v_i$ (with $0 \leq v_L < v_H < \infty$). The bidders' individual private valuations are drawn from a common continuous uniform distribution function $F(v) = \frac{v - v_L}{v_H - v_L}$ (such that $F(v_L) = 0$ and $F(v_H) = 1$) with an associated probability density function $f(v)$ with $f(v) = \frac{\partial F(v)}{\partial v} = \frac{1}{v_H - v_L} \in (0, \infty)$. If a bidder with valuation $v_i$ obtains the good in the auction at price $p$, his payoff is $v_i - p$; if he does not successfully participate in the auction or does not submit a bid, he obtains a payoff of zero. On the other hand, a risk averse seller with a strictly concave continuous utility function $u(x)$ for a given realization of revenue $x$ (such that $u(0) = 0$, $\frac{\partial u(x)}{\partial x} > 0$ and $\frac{\partial^2 u(x)}{\partial x^2} < 0$ for all $x > 0$) offers two homogeneous goods for sale.[2] Further, assume that $u(x)$ takes the form $\sqrt{x}$.[3]

[1] The particular pricing rule will be described in more detail later on in this chapter.

[2] Given a distribution of realizations of revenue $R(x)$, the expected utility for such a seller is $U(R(x)) = \int u(x)\, dR(x)$.

[3] Clearly, the necessary conditions to characterize risk aversion hold for $u(x) = \sqrt{x}$ since $\frac{\partial u(x)}{\partial x} = \frac{1}{2\sqrt{x}} > 0$ and $\frac{\partial^2 u(x)}{\partial x^2} = -\frac{1}{4x^{\frac{3}{2}}} < 0$ for any $x > 0$.

The model is set up as a three stage sequential game, depicted in Figure 5.1:

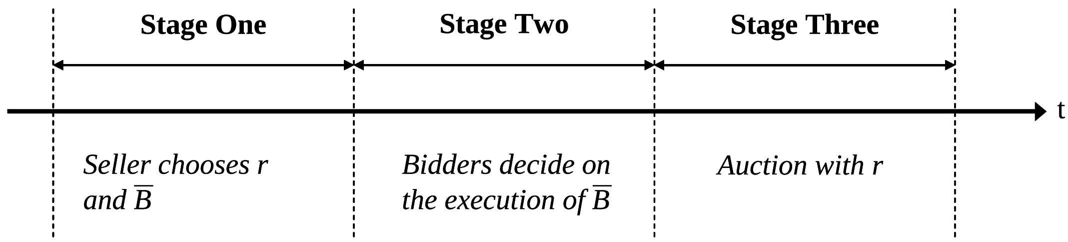


Figure 5.1: Course of the game

In the **first stage** of the game, the seller chooses a Buyout Price $\overline{B}$ along with a reserve price $r$, both identically applying for the two respective homogeneous goods at sale. Obviously, a seller will choose $\overline{B} \geq r$ since otherwise he would make a fixed price offer at a price lower than his reserve price.

In the **second stage** of the game, all bidders simultaneously and independently decide on the execution of the Buyout Option at the ex ante specified Buyout Price $\overline{B}$. If multiple bidders exercise the Buyout Option, the goods are being awarded randomly with each of these bidders having the same probability of receiving a good.[4] If both goods are sold in the second stage, the game ends and no further supply is being tendered in the auction in stage three. If the game does not come to an end after stage two, the Buyout Option expires and is no longer available to the bidders in the subsequent stage.

If supply exceeds demand after stage two, that is, if no bidder exercises the Buyout Option or only a single bidder exercises the Buyout Option respectively, the remaining goods are being sold by way of a multi unit Vickrey auction with discriminatory pricing with reserve price $r$ in **stage three** of the game. That is, an auction only takes place if not all goods have been sold in the second stage of the game (i.e. if at least one good is still at sale). It is further assumed that bidders can only submit an identical bid for both goods, that is, they cannot bid different prices for the two goods.[5] Thus, the bid they submit for either the Buyout Option or the auction identically applies to all available goods.

Throughout it is being assumed that the exogenously given number of potential bidders in the auction, the distribution of bidders' individual valuations and the

[4] If $n$ bidders exercise the Buyout Option, each of the $n$ bidders receives a good with probability $\frac{2}{n}$ (with $n > 2$). If only one or two bidders exercise the Buyout Option, they receive the good with probability 1.

[5] Throughout the discussion of the model, bids are expressed in terms of prices.

seller's reserve price $r$ and Buyout Price $\overline{B}$ are common knowledge. The information asymmetry in the model therefore stems from the fact that each bidder knows his own valuation with certainty but only knows the distribution from which the other bidders' valuations are drawn and the seller only knows the distribution of the bidder's valuations.

A special clarification is needed for the pricing rule of the goods in the model. In the second stage of the game the seller makes a fixed price offer at the ex ante prespecified Buyout Price $\overline{B} \geq r$. Thus, the fully disclosed Buyout Price does not change over the course of the second stage of the game and applies for both goods alike. However, in stage three of the game where the Buyout Option has already expired, a discriminatory pricing mechanism is employed. The prices for the goods auctioned in stage three of the game can potentially vary, depending on the bids submitted.

If a single good is sold in the second stage of the game (if only a single bidder exercises the Buyout Option), the price for the remaining good in stage three of the game will be determined following a standard single unit second price sealed bid auction.[6] That is, if the highest and second highest bids in the auction both exceed the reserve price, the bidder with the highest bid receives the good paying the second highest bid (or equivalently the highest losing bid). If the highest bid exceeds the reserve price and the second highest bid is lower than the reserve price (or equal to it), the bidder submitting the highest bid receives the good and pays the reserve price. If no bidder bids above the reserve price, the good is not sold and all bidders and the seller end up receiving a payoff of zero.

If however no bidder exercises the Buyout Option in stage two and both goods are being sold in the auction, the prices for the goods in stage three are determined as follows: Denote by $b^1$ the highest bid, by $b^2$ the second highest bid and by $b^3$ the third highest bid submitted in the auction and by $p^1$ and $p^2$ the prices paid for the first and second good respectively. If $b^1 > r$, $b^2 > r$ and $b^3 > r$, the price for the first good is $p^1 = b^2$ and the price for the second good is $p^2 = b^3$. Thus, in this case both prices for the goods exceed the reserve price (i.e., $p^1 > r$ and $p^2 > r$). If $b^1 > r$, $b^2 > r$ and $b^3 \leq r$, the price for the first good is $p^1 = b^2$ while the price for the second good is $p^2 = r$. If $b^1 > r$, $b^2 \leq r$ and $b^3 \leq r$, it will follow that $p^1 = r$ and $p^2 = r$ if $b^2 = r$. If however $b^2 < r$, the second good will not be sold and the seller receives a payoff of zero from the second good. If $b^1 \leq r$, $b^2 \leq r$ and $b^3 \leq r$ it

6 Note that if only a single good is being offered, the multi unit Vickrey auction reduces to a standard single unit second price auction, see section 2.5 of this thesis.

will analogously follow that $p^1 = r$ in the event of $b^1 = r$ and $p^2 = r$ if $b^2 = r$. The seller will receive a payoff of zero if both $b^1$ and $b^2$ are less than the reserve price since in that case, no good is sold in the auction.

The expected prices $p^1$ and $p^2$ paid in the auction if two goods are at sale can thus be summarized as follows:

$$E\left[p^1\right] = \begin{cases} b^2 & \text{if } b^1 > r \text{ and } b^2 > r \\ r & \text{if } b^1 \geq r \text{ and } b^2 \leq r \\ 0 & \text{if } b^1 < r \end{cases}$$

and

$$E\left[p^2\right] = \begin{cases} b^3 & \text{if } b^1 > r,\ b^2 > r \text{ and } b^3 > r \\ r & \text{if } b^1 \geq r,\ b^2 \geq r \text{ and } b^3 \leq r \\ 0 & \text{if } b^2 < r \end{cases}$$

Thus, if both goods are sold in stage two of the game, the seller's utility is given by $u\left(2\overline{B}\right)$. If one good is sold in stage two and one in stage three at a price $\widetilde{p} \geq r$ his utility is $u\left(\overline{B} + \widetilde{p}\right)$. If only one good is sold in stage two and none in stage three, his utility simply is $u\left(\overline{B}\right)$. If however both units are sold in stage three at a price $\widetilde{p}^1 \geq r$ for the first good and at a price $\widetilde{p}^2 \geq r$ for the second good, his utility is $u\left(\widetilde{p}^1 + \widetilde{p}^2\right)$. If only one good is sold in stage three at a price $\widetilde{p}^3 \geq r$, his utility is $u\left(\widetilde{p}^3\right)$, and if no good is sold either in stage two or three, his utility is zero. The exact pricing mechanism for the auction will be explained in more detail when turning to the analysis of expected seller utility.

It has thereby been shown that in the auction, the winning bidders do not necessarily pay the same price for the goods and that the bidder with the highest valuation ends up paying a surplus compared to the price paid by the bidder with the second highest valuation for the good with positive probability (that is, if the bidder submitting the second highest bid has a valuation above the reserve price). However, since each bidder features single unit demand, he has no incentive to lower his bid as it could occur if he would feature multi unit demand.[7]

Since the decisions of the seller and the bidders are being made sequentially, the game can be solved recursively, starting with the bidder's decision in the second and third stage of the game on whether to execute the Buyout Option or participating in the auction instead. Based on the findings from the analysis of equilibrium bidder behaviour, the decisions on the choice of a reserve price and Buyout Price faced by the seller in stage one of the game can then subsequently be examined.

## 5.2 Optimal Equilibrium Bidder Behaviour

Since the model can be solved recursively by way of backward induction, in a first step optimal bidder behaviour in the second and third stage of the game is analyzed. To characterize optimal bidder behaviour in the model, a symmetric equilibrium in which each bidder follows a pure strategy needs to be computed.

Clearly only bidders with $v_i \geq r$ will ever consider participating in the auction. If a bidder with $v_i < r$ were to participate in the auction, he could receive a negative payoff ($v_i - r < 0$ for all $v_i < r$) since, upon successful participation, he would be paying a price exceeding his valuation with positive probability depending on the next highest bid.[8] Similarly, only bidders with a valuation $v_i \geq \overline{B}$ will ever consider exercising the Buyout Option in stage two since otherwise they would receive a strictly negative payoff with positive probability ($v_i - \overline{B} < 0$ for all $v_i < \overline{B}$). It therefore remains to specify a pure equilibrium strategy for those bidders with $v_i \geq r$ that determines under which circumstances they will exercise the Buyout Option or rather participate in the auction in stage three. A specific threshold valuation $\overline{v}$ needs to be found that determines optimal equilibrium bidder behaviour, that is, a threshold value that exactly states which bidders would exercise the Buyout Option or participate in the auction instead (where bidders with $v_i < \overline{v}$ would prefer to bid in the auction, bidders with $v_i = \overline{v}$ would be indifferent and bidders with $v_i > \overline{v}$ would exercise the Buyout Option).

As it has been shown by Vickrey (1961) in his seminal paper in auction theory, it is optimal for a bidder to truthfully bid his own valuation in a standard single unit

[7] Indeed with multi unit demand, bidders may shade their bids, see section 2.5 of this thesis.

[8] Note that at the same time, only bids at least as high as the reserve price are deemed adequate by the seller. Thus, submitting a bid strictly lower than the reserve price has no effect on the outcome of the auction.

second price sealed bid auction. Assume for an auction where individual bidder's payoffs are defined as $v - p$ that a bidder $i$ with valuation $v_i \geq r$ would instead submit a bid higher than his valuation $v_i^+ > v_i$.[9] If he would win the auction (if he were the bidder with the highest bid submitted) and the next highest bid $v_{-i}^{\max} < v_i^+$ (the highest losing bid) would exceed bidder $i$'s true valuation, that is $v_{-i}^{\max} > v_i$, he would have to pay a price $p = v_{-i}^{\max} > v_i$ guaranteeing him a strictly negative payoff ($v_i - p < 0$). If instead the bidder would submit a bid lower than his valuation $v^- < v_i$ and at the same time some other bidder would submit a bid $v_{-i}^*$ with $v_{-i}^* > v^-$ and $v_{-i}^* < v_i$, the bidder would not win the good and end up receiving a payoff of zero when he could instead have overbid the highest bidder and still gaining a positive payoff, that is, if bidder $i$ had instead submitted a bid $v_{-i}^* + \varepsilon < v_i$ (with $\varepsilon > 0$). Bidders in single unit second price sealed bid auctions therefore have no incentive to deviate from truthful bidding as this guarantees them the highest possible expected payoff. It is thus a dominant strategy for bidders in a single unit second price sealed bid auction to bid their true valuation. The same holds for a multi unit second price sealed bid auction or Vickrey auction with single unit demand. In this setup, it is just as well a dominant strategy for every bidder to truthfully bid in the auction since this will guarantee them the highest possible expected payoff.[10] If every bidder follows this strategy, the outcome of the auction is efficient in that the bidders with the highest valuations receive the goods. Thereby, optimal bidder behaviour for stage three of the game is given, that is, every bidder will bid his true valuation for the good.

**Expected Bidder Payoff from not Exercising the Buyout Option**

To start with, the expected payoff for a bidder who does not exercise the Buyout Option in stage two of the game and only participates in the auction in stage three of the game is derived. For any arbitrary bidder with valuation $v_i$, the expected payoff from *not exercising* the Buyout Option in stage two and solely participating in the auction in stage three is:[11]

[9] As it has been shown, only bidders with $v_i \geq r$ will ever participate in the auction.
[10] See Vickrey (1961).
[11] Recall that only bidders with a valuation at or above the reserve price participate in the auction.

$$\begin{aligned}\lambda(v_i) &= (v_i - r) F(r)^n \\ &+ \int_r^{\min\{v_i,\overline{v}\}} (v_i - y) nF(y)^{n-1} f(y) \, dy \\ &+ (v_i - r) F(r)^{n-1} (1 - F(v_i)) \\ &+ \int_r^{\min\{v_i,\overline{v}\}} (v_i - y)(n-1) F(y)^{n-2} f(y)(1 - F(v_i)) \, dy,\end{aligned}$$

where the first two terms combined are his expected payoff if he were the bidder with the highest valuation whilst the third and fourth term combined describe his expected payoff if he were the bidder with the second highest valuation, given that all bidders follow the dominant strategy of bidding their true valuation. That is, if any given bidder with $v_i$ were the bidder with the highest valuation and the second highest bidder's valuation were $v_{-i}^{\max} \leq r$, he would receive a payoff of $v_i - r$. If the second highest bidder's valuation would though exceed the reserve price, bidder $i$ would end up paying a price higher than the reserve price. If however bidder $i$ were the bidder with the second highest valuation, he would again pay a price equal to the reserve price if the next highest bid would be lower than or equal to $r$ and a price equal to the third highest bid if it exceeded $r$. If bidder $i$ had a valuation lower than the second highest valuation he would not win a good in the auction and receive a payoff of zero. It is further assumed that bidders competing in the auction do not have additional participation costs.

$\lambda(v_i)$ can be simplified and rewritten as:[12]

$$\begin{aligned}\lambda(v_i) &= (v_i - \min\{v_i,\overline{v}\}) \left[F(\min\{v_i,\overline{v}\})^n + (1 - F(v_i)) F(\min\{v_i,\overline{v}\})^{n-1}\right] \\ &+ \int_r^{\min\{v_i,\overline{v}\}} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy.\end{aligned}$$

[12] See Appendix $A$.1.

**Expected Bidder Payoff from Exercising the Buyout Option**

Having derived bidders' expected payoff from only actively participating in the auction in stage three, the expected payoff of a bidder with $v_i$ from *exercising* the Buyout Option in stage two of the game can analogously be derived. Such a bidder's expected payoff is:

$$\gamma\left(v_i\right)=\left(v_i-\overline{B}\right)\left[2\sum_{j=0}^{n}\frac{\left(1-F\left(\overline{v}\right)\right)^j F\left(\overline{v}\right)^{n-j}}{1+j}\binom{n}{j}-F\left(\overline{v}\right)^n\right],$$

where the first multiplier is his payoff (the difference between his valuation and the Buyout Price) and the second multiplier is the probability with which he is awarded a good upon exercising the Buyout Option.[13] This expression can be simplified to:[14]

$$\gamma\left(v_i\right)=\left(v_i-\overline{B}\right)\frac{2\left(1-F\left(\overline{v}\right)^{n+1}\right)-\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^n}{\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)}.$$

Hence, the bidder's expected payoffs when either participating in the second or third stage of the game have been explicitly determined. In what follows, optimal equilibrium bidder behaviour is being derived on the basis of a consideration of these distinctive expected payoffs.

**Equilibrium Bidder Threshold Buyout Price**

To characterize optimal equilibrium bidder behaviour, the expected payoff from exercising the Buyout Option needs to be compared with the expected payoff from not exercising the Buyout Option and participating in the auction instead. Consider therefore an equilibrium bidder with valuation $v_i=\overline{v}$. For such a bidder for a strategy to characterize an equilibrium, he must receive the same expected payoff from exercising or not exercising the Buyout Option. Therefore, the following condition must hold in equilibrium:

$$\lambda\left(\overline{v}\right)=\gamma\left(\overline{v}\right).$$

[13] The probability of successfully exercising the Buyout Option is the probability of receiving the good upon execution given the probabilities of other bidders simultaneously exercising the Buyout Option.

[14] See Appendix $A$.2.

Since

$$\lambda(\overline{v}) = \int_{r}^{\overline{v}} F(y)^n + (1 - F(\overline{v})) F(y)^{n-1} \, dy$$

and

$$\gamma(\overline{v}) = \left(\overline{v} - \overline{B}\right) \frac{2\left(1 - F(\overline{v})^{n+1}\right) - (n+1)(1 - F(\overline{v})) F(\overline{v})^n}{(n+1)(1 - F(\overline{v}))},$$

$\lambda(\overline{v}) = \gamma(\overline{v})$ can equivalently be expressed as

$$\begin{aligned} & \int_{r}^{\overline{v}} F(y)^n + (1 - F(\overline{v})) F(y)^{n-1} \, dy \\ = \quad & \left(\overline{v} - \overline{B}\right) \frac{2\left(1 - F(\overline{v})^{n+1}\right) - (n+1)(1 - F(\overline{v})) F(\overline{v})^n}{(n+1)(1 - F(\overline{v}))}. \end{aligned}$$

From this equilibrium condition, for a bidder with valuation $v_i = \overline{v}$, the equilibrium threshold Buyout Price that specifies optimal bidding behaviour can be explicitly derived:

$$\overline{B}(\overline{v}, r) = \overline{v} - \left[ \begin{array}{c} \frac{(n+1)(1-F(\overline{v}))}{2\left(1-F(\overline{v})^{n+1}\right)-(n+1)(1-F(\overline{v}))F(\overline{v})^n} \\ \times \left\{ \int_{r}^{\overline{v}} F(y)^n + (1 - F(\overline{v})) F(y)^{n-1} \, dy \right\} \end{array} \right].$$

Therefore, for a bidder with $v_i = \overline{v}$, if the seller sets a Buyout Price lower than $\overline{B}(\overline{v}, r)$, he will exercise the Buyout Option in stage two of the game. If however the ex ante specified Buyout Price is set equal to or above his threshold value ($\overline{B} \geq \overline{B}(\overline{v}, r)$), he will not exercise the Buyout Option but only participate in the auction in stage three instead.[15]

[15] It is assumed that even though such a bidder is effectively indifferent between exercising and not exercising the Buyout Option if $\overline{B} = \overline{B}(\overline{v}, r)$, he will in that case not exercise the Buyout Option.

To further determine optimal equilibrium bidder behaviour for any given bidder with a valuation $v_i \neq \overline{v}$, the respective marginal expected payoffs need to be taken into account. The marginal expected payoff from not exercising the Buyout Option for a bidder with $v_i$ is:[16]

$$\begin{aligned}\frac{\partial \lambda(v_i)}{\partial v_i} &= F(\min\{v_i, \overline{v}\})^n \\ &\quad + (1 - F(v_i)) F(\min\{v_i, \overline{v}\})^{n-1} \\ &\quad - (v_i - \min\{v_i, \overline{v}\}) F(\min\{v_i, \overline{v}\})^{n-1} f(v_i) \\ &\quad - \int_r^{\min\{v_i, \overline{v}\}} f(v_i) F(y)^{n-1} dy.\end{aligned}$$

Note that

$$\left.\frac{\partial \lambda(v_i)}{\partial v_i}\right|_{v_i < \overline{v}} = F(v_i)^n + (1 - F(v_i)) F(v_i)^{n-1} - \int_r^{v_i} f(v_i) F(y)^{n-1} dy$$

and

$$\left.\frac{\partial \lambda(v_i)}{\partial v_i}\right|_{v_i = \overline{v}} = F(\overline{v})^n + (1 - F(\overline{v})) F(\overline{v})^{n-1} - \int_r^{\overline{v}} f(\overline{v}) F(y)^{n-1} dy,$$

from which it follows that $\left.\frac{\partial \lambda(v_i)}{\partial v_i}\right|_{v_i < \overline{v}} < \left.\frac{\partial \lambda(v_i)}{\partial v_i}\right|_{v_i = \overline{v}}$.[17]

On the other hand, for such a bidder, his marginal expected payoff from exercising the Buyout Option is

$$\frac{\partial \gamma(v_i)}{\partial v_i} = \frac{2\left(1 - F(\overline{v})^{n+1}\right) - (n+1)(1 - F(\overline{v})) F(\overline{v})^n}{(n+1)(1 - F(\overline{v}))}.$$

[16] See Appendix $A$.3.

[17] See Appendix $B$.1.

For a bidder with valuation $v_i > \overline{v}$, in equilibrium his marginal expected payoff from exercising the Buyout Option must exceed his marginal expected payoff from not exercising the Buyout Option for him to exercise the Buyout Option. Again the case for the equilibrium bidder with $v_i = \overline{v}$ is examined, that is $\left.\frac{\partial\gamma(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} > \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$ must hold.

Thus for such a bidder, the following condition must be fulfilled:

$$\begin{aligned} & \frac{2\left(1-F\left(\overline{v}\right)^{n+1}\right)-\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n}}{\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)} \\ > \quad & F\left(\overline{v}\right)^{n}+\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n-1}-\int_{r}^{\overline{v}} f\left(\overline{v}\right)F\left(y\right)^{n-1}dy. \end{aligned}$$

This condition clearly holds for all $F\left(\overline{v}\right) < 1$.[18] It therefore follows from $\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}} < \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$ and $\left.\frac{\partial\gamma(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} > \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$ that $\gamma\left(v_i\right) > \lambda\left(v_i\right)$ for any bidder with $v_i > \overline{v}$ and $\gamma\left(v_i\right) < \lambda\left(v_i\right)$ for all bidders with $v_i < \overline{v}$. Thus, a bidder with a valuation $v_i > \overline{v}$ has a higher expected payoff from exercising the Buyout Option while a bidder with $v_i < \overline{v}$ has a higher expected payoff from not exercising the Buyout Option and only participating in the auction in stage three. Thereby the equilibrium threshold value $\overline{v}$ has been explicitly defined that characterizes optimal bidder behaviour.

From this, the threshold equilibrium Buyout Price $B\left(v_i, r\right)$ for any bidder with $v_i$ can be derived and is given by:

$$B\left(v_i, r\right) = v_i - \left[\begin{array}{c} \frac{(n+1)(1-F(v_i))}{2\left(1-F(v_i)^{n+1}\right)-(n+1)(1-F(v_i))F(v_i)^{n}} \\ \times\left\{\int_{r}^{v_i} F\left(y\right)^{n}+\left(1-F\left(v_i\right)\right)F\left(y\right)^{n-1}dy\right\} \end{array}\right].$$

The equilibrium strategies for all bidders with $v_i \in \left[v_L, v_H\right]$ can therefore be described as follows: Denote a bidder's equilibrium strategy space by $S\left(v_i\right)_b^a$ where the superscript $a \in \{\text{bid}, \text{no bid}\}$ describes bidding behaviour for the execution of the Buyout Option in stage two (where "bid" implies exercising the Buyout Option and "no bid" implies forfeiting the Buyout Option) and the subscript $b \in \{\text{bid } v_i, \text{no bid}\}$

[18] See Appendix $B.2$.

describes bidding behaviour for stage three of the game (where "bid" implies truthfully bidding his valuation and "no bid" implies not submitting any bid and thereby not participating in the auction). Formally the optimal equilibrium strategy $\widehat{S}(v_i)$ for a bidder with a valuation $v_i$ is of the form:

$$\widehat{S}(v_i)\begin{cases} S(v_i)^{\text{bid}}_{\text{bid } v_i} & \text{if } v_i \geq r \text{ and } B(v_i, r) > \overline{B} \\ S(v_i)^{\text{no bid}}_{\text{bid } v_i} & \text{if } v_i \geq r \text{ and } B(v_i, r) \leq \overline{B} \\ S(v_i)^{\text{no bid}}_{\text{no bid}} & \text{if } v_i < r \text{ and } B(v_i, r) \leq \overline{B}. \end{cases}$$

$\widehat{S}(v_i)$ thus implies that if in stage one of the game the seller would choose a Buyout Price $\overline{B} \geq B(v_i, r)$, a bidder with valuation $v_i$ will never exercise the Buyout Option. In this case, such a bidder will instead participate in the auction in stage three so long as $v_i \geq r$. If however the seller would choose to set a Buyout Price $\overline{B} < B(v_i, r)$, a bidder with $v_i$ would exercise the Buyout Option in equilibrium and waive his participation in the auction.[19] If the seller would set the Buyout Price at $\overline{B} = B(r, r)$ every bidder with $v_i > r$ would exercise the Buyout Option (since $B(r, r) = r$). Furthermore, if the seller chooses a Buyout Price $\overline{B} \geq B(v_H, r)$ no bidder will exercise the Option and both goods would be sold by way of a multi unit Vickrey auction in stage three of the game. This strategy space determines a symmetric equilibrium since no bidder has an incentive to deviate as it will be shown hereafter.

To see that the derived equilibrium threshold Buyout Price for any bidder with $v_i$ does indeed characterize a unique symmetric equilibrium from the strategy space $\widehat{S}(v_i)$, it must be shown that there exists no other stable equilibrium. Assume that there was another equilibrium characterized by a threshold value $\widetilde{v} \neq \overline{v}$. If $\widetilde{v}$ would indeed constitute another stable equilibrium, a bidder with $v_i = \widetilde{v}$ would need to have the same expected payoff from both exercising and not exercising the Buyout Option. This would imply that

[19] Such a bidder would nevertheless post a bid for the auction since by executing the Buyout Option he does not necessarily get one of the goods with probability one. However, this would not have any impact on the outcome of the auction since if a bidder indeed exercising the Buyout Option does not receive a good, the game ends.

$$\int_r^{\widetilde{v}} F(y)^n + (1 - F(\widetilde{v}))\, F(y)^{n-1}\, dy$$

$$= \quad \left(\widetilde{v} - \overline{B}\right) \frac{2\left(1 - F(\widetilde{v})^{n+1}\right) - (n+1)\,(1 - F(\widetilde{v}))\, F(\widetilde{v})^n}{(n+1)\,(1 - F(\widetilde{v}))}$$

and thus $B(\widetilde{v}, r) = \widetilde{v} - \left[\begin{array}{c} \frac{(n+1)(1-F(\widetilde{v}))}{2\left(1-F(\widetilde{v})^{n+1}\right)-(n+1)(1-F(\widetilde{v}))F(\widetilde{v})^n} \\ \left\{\int_r^{\widetilde{v}} F(y)^n + (1 - F(\widetilde{v}))\, F(y)^{n-1}\, dy\right\} \end{array}\right]$ or analogously that $B(\widetilde{v}, r) = \overline{B} = B(\overline{v}, r)$ (since $\overline{v}$ is characterized by $\overline{B} = B(\overline{v}, r)$), which cannot hold due to the fact that the threshold Buyout Price is strictly increasing in $v$ $\left(\frac{\partial B(v,r)}{\partial v} > 0\right)$.[20] Therefore $B(v, r) = v - \left[\begin{array}{c} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \left\{\int_r^{v} F(y)^n + (1 - F(v))\, F(y)^{n-1}\, dy\right\} \end{array}\right]$ supports a unique symmetric equilibrium from the set of optimal strategies.

**Comparative Statics of the Threshold Buyout Price**

To gain further insight into optimal bidder behaviour, the ceteris paribus impact of a change in either the underlying private valuation or the reserve price is being analyzed.

The first derivative of the threshold Buyout Price over a bidder's private valuation yields

$$\begin{aligned} \frac{\partial B(v, r)}{\partial v} \quad &= \quad 1 - \left[\begin{array}{c} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left[F(v)^{n-1} - \int_r^{v} f(v)\, F(y)^{n-1}\, dy\right] \end{array}\right] \\ & \quad + \left[\begin{array}{c} \frac{(n+1)f(v)}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left\{1 - \frac{(n+1)(1-F(v))F(v)^{n-1}[F(v)+(1-F(v))n]}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n}\right\} \\ \times \left\{\int_r^{v} F(y)^n + (1 - F(v))\, F(y)^{n-1}\, dy\right\} \end{array}\right], \end{aligned}$$

[20] See Appendix $B.3$.

which is strictly positive for all $F(v) < 1$.[21] Thus for such bidders (i.e. a bidder with $v_i \in [v_L, v_H)$), the higher a bidder's valuation the higher is his threshold Buyout Price. This seems quite obvious since the higher a bidder values the good, the sooner he will exercise the Buyout Option, that is, he would accept a higher Buyout Price than if he had a lower valuation for the good. This points to the fact that if a bidder has a relatively low valuation, his incentive to participate in the auction instead of exercising the Buyout Option in the second stage of the game is relatively high and such bidders would accept to bear the risk of not winning the auction even if their valuation might exceed the posted Buyout Price and despite the fact that they could have gained a positive payoff when exercising the Buyout Option with positive probability. A bidder with a comparably high valuation on the other hand is willing to pay a higher price for the good and therefore sooner exercise the Buyout Option.

Likewise, the first derivative of the threshold Buyout Price over the reserve price is

$$\begin{aligned}\frac{\partial B(v,r)}{\partial r} &= \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} \\ &\quad \times \left[F(r)^{n}+(1-F(v))F(r)^{n-1}\right],\end{aligned}$$

which is strictly positive for all $F(v) < 1$.[22] Therefore, if the seller sets a relatively high reserve price, the Buyout Option is sooner exercised by those bidders with a valuation still exceeding the prevailing reserve price. This results from the fact that a successful participation in the auction in stage three of the game becomes potentially more costly if the seller increases the reserve price (as a bidder will end up paying more for the good if the price in the auction is indeed determined by the reserve price).

It has therefore been shown that the threshold Buyout Price is increasing in both the individual bidder's valuation and the reserve price posted by the seller (i.e., $\frac{\partial B(v,r)}{\partial v} > 0$ and $\frac{\partial B(v,r)}{\partial r} > 0$ for $F(v) < 1$).

[21] See Appendix $B.3$.
[22] See Appendix $B.4$.

## 5.3 Optimal Equilibrium Seller Behaviour

Given optimal equilibrium bidder behaviour in stage two and three of the game, the choice of $\overline{B}$ and $r$ by the seller in stage one will now be derived. Since $\overline{B}$ depends on the two variables $\overline{v}$ and $r$, the seller's optimization problem needs to be analyzed as an optimal choice of $\overline{v}$ and $r$. Given that the seller simultaneously offers two homogeneous goods for sale, the expected joint utility from each of the individual goods needs to be considered when computing overall expected seller utility.

### Optimal Choice of $\overline{B}$

Given that all bidders follow the equilibrium strategy described above, the expected joint utility for a seller from offering the two goods is

$$
\begin{aligned}
U(\overline{v}, r) = & \int_{\overline{v}}^{v_H} \left\{ \begin{array}{l} \int_{\overline{v}}^{y} u\left(2B\left(\overline{v}, r\right)\right) k(z)\, dz \\ \\ +\int_{r}^{\overline{v}} \left\{ \begin{array}{l} \int_{r}^{z} u\left(B\left(\overline{v}, r\right) + q\right) l(q)\, dq \\ +\int_{v_L}^{r} u\left(B\left(\overline{v}, r\right) + r\right) l(q)\, dq \end{array} \right\} k(z)\, dz \\ \\ +\int_{v_L}^{r} u\left(B\left(\overline{v}, r\right)\right) k(z)\, dz \end{array} \right\} h(y)\, dy \\
& + \int_{r}^{\overline{v}} \left\{ \begin{array}{l} \int_{r}^{y} \left\{ \begin{array}{l} \int_{r}^{z} u(z+q)\, l(q)\, dq \\ +\int_{v_L}^{r} u(z+r)\, l(q)\, dq \end{array} \right\} k(z)\, dz \\ \\ +\int_{v_L}^{r} u(r)\, k(z)\, dz \end{array} \right\} h(y)\, dy,
\end{aligned}
$$

where $y$ is the largest, $z$ is the second largest and $q$ is the third largest of the $n+1$ realized bidder valuations. $h(y)$ denotes the probability density function of $y$ while $k(z)$ denotes the probability density function of $z$ given $y$ and $l(q)$ the probability density function of $q$ given $y$ and $z$. The first term describes the seller's expected utility if the highest of the $n+1$ realized $v_i$ is above the equilibrium threshold valuation ($y > \overline{v}$). As it has been examined from equilibrium bidder behaviour, a bidder with $v_i = y$ will then exercise the Buyout Option. If $z$ as well lies above the equilibrium threshold value of $\overline{v}$, the seller's utility would accordingly be $u\left(2B\left(\overline{v}, r\right)\right)$. However, if the second highest bidder's valuation is between the reserve price and the equilibrium threshold value, the seller's utility would either be $u\left(B\left(\overline{v}, r\right) + q\right)$ if the third highest bidder's valuation exceeds the reserve price or $u\left(B\left(\overline{v}, r\right) + r\right)$ if $q \leq r$. If the second highest valuation is below the reserve price, his utility simply is $u\left(B\left(\overline{v}, r\right)\right)$ since only a single good will be sold at the Buyout Price. The second term of the sum describes the seller's expected utility if the highest bidder's valuation is at or above the reserve price but lower than the equilibrium threshold value (in that case, even the bidder with the highest valuation would not exercise the Buyout Option). If $z$ exceeds the reserve price, the seller's utility is either $u\left(z + q\right)$ if $q > r$ or $u\left(z + r\right)$ if $q \leq r$, that is, the seller would receive either an amount equal to the second and third highest valuation or the second highest valuation and the reserve price. Finally, if only the highest bidder's valuation exceeds or is equal to the reserve price, the seller would receive $r$. If $y < r$, the seller receives zero. The seller's expected utility $E\left[U\left(\overline{v}, r\right)\right]$ can therefore be summarized as follows:

$$E\left[U\left(\overline{v}, r\right)\right] = \begin{cases} u\left(2\overline{B}\right) & \text{if } y \in \left(\overline{v}, v_H\right] \text{ and } z \in \left(\overline{v}, y\right] \\ u\left(\overline{B} + q\right) & \text{if } y \in \left(\overline{v}, v_H\right], \, z \in \left[r, \overline{v}\right] \text{ and } q \in \left[r, z\right] \\ u\left(\overline{B} + r\right) & \text{if } y \in \left(\overline{v}, v_H\right], \, z \in \left[r, \overline{v}\right] \text{ and } q \in \left[v_L, r\right) \\ u\left(\overline{B}\right) & \text{if } y \in \left(\overline{v}, v_H\right] \text{ and } z \in \left[v_L, r\right) \\ u\left(z + q\right) & \text{if } y \in \left[r, \overline{v}\right], \, z \in \left[r, y\right] \text{ and } q \in \left[r, z\right] \\ u\left(z + r\right) & \text{if } y \in \left[r, \overline{v}\right], \, z \in \left[r, y\right] \text{ and } q \in \left[v_L, r\right) \\ u\left(r\right) & \text{if } y \in \left[r, \overline{v}\right] \text{ and } z \in \left[v_L, r\right) \\ 0 & \text{if } y \in \left[v_L, r\right). \end{cases}$$

Substituting for $h(y) = (n+1) F(y)^n f(y)$, $k(z) = \frac{nF(z)^{n-1} f(z)}{F(y)^n}$ and $l(q) = \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}}$, and further simplifying for $F(v_L) = 0$ and $F(v_H) = 1$, $U(\overline{v}, r)$ can be rewritten as:[23]

$$
\begin{aligned}
U(\overline{v}, r) &= u(2B(\overline{v}, r)) \int_{\overline{v}}^{v_H} n(n+1) F(z)^{n-1} f(z) [1 - F(z)] \, dz \\
&+ \int_{r}^{\overline{v}} \left[ \begin{array}{l} u(B(\overline{v}, r) + q) n(n+1)(n-1) \\ \times F(q)^{n-2} f(q) [F(\overline{v}) - F(q)] [1 - F(\overline{v})] \end{array} \right] dq \\
&+ u(B(\overline{v}, r) + r) n(n+1) F(r)^{n-1} [F(\overline{v}) - F(r)] [1 - F(\overline{v})] \\
&+ u(B(\overline{v}, r)) (n+1) F(r)^n [1 - F(\overline{v})] \\
&+ \int_{r}^{\overline{v}} \left\{ \int_{r}^{z} \left[ \begin{array}{l} u(z+q) n(n+1) \\ \times (n-1) F(q)^{n-2} f(q) \end{array} \right] dq \right\} f(z) [F(\overline{v}) - F(z)] \, dz \\
&+ \int_{r}^{\overline{v}} u(z+r) n(n+1) F(r)^{n-1} f(z) [F(\overline{v}) - F(z)] \, dz \\
&+ u(r)(n+1) F(r)^n [F(\overline{v}) - F(r)] .
\end{aligned}
$$

It is straightforward that a seller will choose a Buyout Price $\overline{B} > r$ (or $\overline{v} > r$ since the analysis of optimal seller behaviour is in terms of choosing $\overline{v}$) because otherwise he would offer the goods at a fixed price less than or equal to the reserve price, making the reserve price redundant (since $B(r, r) = r$). If the seller were to choose a Buyout Price $\overline{B} \geq B(v_H, r)$, no bidder would ever exercise the Buyout Option. If it can be shown that seller's expected marginal utility from the highest type bidder is decreasing in $\overline{v}$ ($\left.\frac{\partial U(\overline{v}, r)}{\partial \overline{v}}\right|_{\overline{v}=v_H} < 0$), it would follow that the seller would optimally choose a Buyout Price strictly lower than the threshold Buyout Price for the bidder with the highest valuation (that is, he would choose $\overline{B} < B(v_H, r)$) since then he could ceteris paribus increase his expected utility when lowering the Buyout Price from $B(v_H, r)$ to some $B(v_H, r) - \varepsilon$ (with $\varepsilon > 0$).

[23] For the derivation of the respective probability density functions, see Appendix $A.4$. For the derivation of $U(\overline{v}, r)$, see Appendix $A.5$.

From the seller's expected utility function it follows that

$$
\begin{aligned}
\frac{\partial U\left(\overline{v},r\right)}{\partial \overline{v}} = & \left[\begin{array}{l} 2\frac{\partial u(2B(\overline{v},r))}{\partial \overline{v}}\frac{\partial B(\overline{v},r)}{\partial \overline{v}} \\ \times \int\limits_{\overline{v}}^{v_H} n\left(n+1\right)F\left(z\right)^{n-1}f\left(z\right)\left[1-F\left(z\right)\right]dz \end{array}\right] \\
& -u\left(2B\left(\overline{v},r\right)\right)n\left(n+1\right)F\left(\overline{v}\right)^{n-1}f\left(\overline{v}\right)\left[1-F\left(\overline{v}\right)\right] \\
& +\int\limits_{r}^{\overline{v}}\left[\begin{array}{l} \frac{\partial u(B(\overline{v},r)+q)}{\partial \overline{v}}\frac{\partial B(\overline{v},r)}{\partial \overline{v}}n\left(n+1\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\left[F\left(\overline{v}\right)-F\left(q\right)\right]\left[1-F\left(\overline{v}\right)\right] \end{array}\right]dq \\
& +\int\limits_{r}^{\overline{v}}\left[\begin{array}{l} u\left(B\left(\overline{v},r\right)+q\right)n\left(n+1\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)f\left(\overline{v}\right)\left\{1-2F\left(\overline{v}\right)+F\left(q\right)\right\} \end{array}\right]dq \\
& +\left[\begin{array}{l} \frac{\partial u(B(\overline{v},r)+r)}{\partial \overline{v}}\frac{\partial B(\overline{v},r)}{\partial \overline{v}}n\left(n+1\right) \\ \times F\left(r\right)^{n-1}\left[F\left(\overline{v}\right)-F\left(r\right)\right]\left[1-F\left(\overline{v}\right)\right] \end{array}\right] \\
& +u\left(B\left(\overline{v},r\right)+r\right)n\left(n+1\right)F\left(r\right)^{n-1}f\left(\overline{v}\right)\left[1-2F\left(\overline{v}\right)+F\left(r\right)\right] \\
& +\frac{\partial u\left(B\left(\overline{v},r\right)\right)}{\partial \overline{v}}\frac{\partial B\left(\overline{v},r\right)}{\partial \overline{v}}\left(n+1\right)F\left(r\right)^{n}\left[1-F\left(\overline{v}\right)\right] \\
& -u\left(B\left(\overline{v},r\right)\right)\left(n+1\right)F\left(r\right)^{n}f\left(\overline{v}\right) \\
& +\int\limits_{r}^{\overline{v}}\left\{\int\limits_{r}^{z}\left[\begin{array}{l} u\left(z+q\right)n\left(n+1\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right) \end{array}\right]dq\right\}f\left(z\right)f\left(\overline{v}\right)dz \\
& +\int\limits_{r}^{\overline{v}}u\left(z+r\right)n\left(n+1\right)F\left(r\right)^{n-1}f\left(z\right)f\left(\overline{v}\right)dz \\
& +u\left(r\right)\left(n+1\right)F\left(r\right)^{n}f\left(\overline{v}\right).
\end{aligned}
$$

With regard to the highest type bidder, his marginal utility is

$$\begin{aligned}
\left.\frac{\partial U\left(\overline{v},r\right)}{\partial \overline{v}}\right|_{\overline{v}=v_H} &= -\int_r^{v_H}\left[\begin{array}{l} u\left(B\left(\overline{v},r\right)+q\right)n\left(n+1\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\left[F\left(v_H\right)-F\left(q\right)\right]f\left(v_H\right)\end{array}\right]dq \\
&\quad -u\left(B\left(\overline{v},r\right)+r\right)n\left(n+1\right)F\left(r\right)^{n-1}\left[F\left(v_H\right)-F\left(r\right)\right]f\left(v_H\right) \\
&\quad -u\left(B\left(\overline{v},r\right)\right)\left(n+1\right)F\left(r\right)^{n}f\left(v_H\right) \\
&\quad +\int_r^{v_H}\left\{\int_r^{z}\left[\begin{array}{l} u\left(z+q\right)n\left(n+1\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\end{array}\right]dq\right\}f\left(z\right)f\left(v_H\right)dz \\
&\quad +\int_r^{v_H}u\left(z+r\right)n\left(n+1\right)F\left(r\right)^{n-1}f\left(z\right)f\left(v_H\right)dz \\
&\quad +u\left(r\right)\left(n+1\right)F\left(r\right)^{n}f\left(v_H\right).
\end{aligned}$$

Rearranging and substituting for $F\left(v_H\right)=1$ yields

$$\left.\frac{\partial U\left(\overline{v},r\right)}{\partial \overline{v}}\right|_{\overline{v}=v_H} = -\left(n+1\right)f\left(v_H\right)\left\{\begin{array}{l}
u\left(B\left(\overline{v},r\right)\right)F\left(r\right)^{n} \\
+\int_r^{v_H}\left[\begin{array}{l} u\left(B\left(\overline{v},r\right)+q\right)n\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\left[1-F\left(q\right)\right]\end{array}\right]dq \\
+u\left(B\left(\overline{v},r\right)+r\right)nF\left(r\right)^{n-1}\left[1-F\left(r\right)\right] \\
-\int_r^{v_H}\left\{\left[\begin{array}{l} \int_r^{z}u\left(z+q\right)n \\ \times\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\end{array}\right]dq\right\}f\left(z\right)dz \\
-\int_r^{v_H}u\left(z+r\right)nF\left(r\right)^{n-1}f\left(z\right)dz \\
-u\left(r\right)F\left(r\right)^{n}
\end{array}\right\}.$$

Define

$$
\begin{aligned}
A(r) &= u\left(B\left(\overline{v},r\right)\right)F(r)^{n} \\
&\quad +\int_{r}^{v_H} u\left(B\left(\overline{v},r\right)+q\right)n\left(n-1\right)F(q)^{n-2}f(q)\left[1-F(q)\right]dq \\
&\quad +u\left(B\left(\overline{v},r\right)+r\right)nF(r)^{n-1}\left[1-F(r)\right] \\
&\quad -\int_{r}^{v_H}\left\{\int_{r}^{z} u\left(z+q\right)n\left(n-1\right)F(q)^{n-2}f(q)\,dq\right\}f(z)\,dz \\
&\quad -\int_{r}^{v_H} u\left(z+r\right)nF(r)^{n-1}f(z)\,dz-u(r)F(r)^{n}.
\end{aligned}
$$

Since $B\left(v_H,r\right)=v_H-\int_{r}^{v_H}F(y)^{n}dy$, it follows that:[24]

$$
\begin{aligned}
A(r) &= u\left(v_H-\int_{r}^{v_H}F(y)^{n}dy\right)F(r)^{n} \\
&\quad +\int_{r}^{v_H} u\left(v_H-\int_{r}^{v_H}F(y)^{n}dy+q\right)n\left(n-1\right)F(q)^{n-2}f(q)\left[1-F(q)\right]dq \\
&\quad +u\left(v_H-\int_{r}^{v_H}F(y)^{n}dy+r\right)nF(r)^{n-1}\left[1-F(r)\right] \\
&\quad -\int_{r}^{v_H}\left\{\int_{r}^{z} u\left(z+q\right)n\left(n-1\right)F(q)^{n-2}f(q)\,dq\right\}f(z)\,dz \\
&\quad -\int_{r}^{v_H} u\left(z+r\right)nF(r)^{n-1}f(z)\,dz-u(r)F(r)^{n}.
\end{aligned}
$$

[24] See Appendix $A.6$.

It suffices to show that $A(r) > 0$ for $\left.\frac{\partial U(\overline{v},r)}{\partial \overline{v}}\right|_{\overline{v}=v_H}$ to be strictly negative for any given value of $v_L \leq r < v_H$. From Figure 5.2 it can be seen that this condition clearly holds:[25]

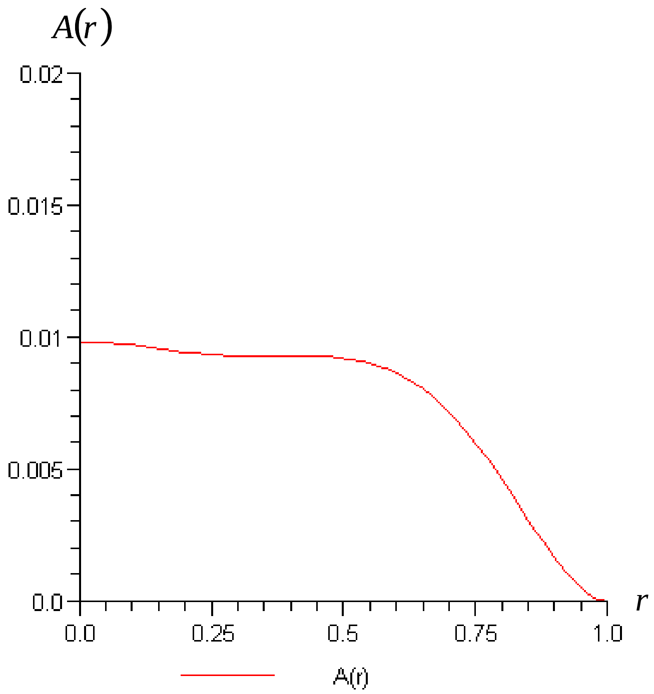


Figure 5.2: $A(r)$ for $n = 3$, $v_H = 1$ and $v_L = 0$

Since $\left.\frac{\partial U(\overline{v},r)}{\partial \overline{v}}\right|_{\overline{v}=v_H} < 0$, the seller will therefore optimally choose a Buyout Price lower than the highest bidder's threshold Buyout Price ($\overline{B} < B(v_H, r)$), regardless of his choice of $r$ since he could thereby strictly increase his expected utility.

**Optimal Choice of $r$**

Furthermore, it can be shown that the seller will optimally choose a reserve price lower than the reserve price he would set if he did not offer a Buyout Option in stage two of the game, that is, if the two goods were auctioned by way of a standard multi unit Vickrey auction.

[25] $A(r) > 0$ in fact holds for any given values of $n$, $v_L$ and $v_H$.

Start by noting that

$$\frac{\partial U(\overline{v},r)}{\partial r}=(n+1)\left\{\begin{array}{l} F(r)^{n-1}f(r)\left\{\begin{array}{l} u(B(\overline{v},r))\,n\,[1-F(\overline{v})]- \\ u(B(\overline{v},r)+r)\,n\,[1-F(\overline{v})] \\ -u(2r)\,n\,[F(\overline{v})-F(r)] \\ +u(r)\{n\,[F(\overline{v})-F(r)]-F(r)\} \end{array}\right\} \\ \\ +\frac{\partial B(\overline{v},r)}{\partial r}\left\{\begin{array}{l} 2\frac{\partial u(2B(\overline{v},r))}{\partial r}\int\limits_{\overline{v}}^{v_H} nF(z)^{n-1}f(z)\,[1-F(z)]\,dz \\ +\int\limits_{r}^{\overline{v}}\left[\begin{array}{l} \frac{\partial u(B(\overline{v},r)+q)}{\partial r}n(n-1) \\ \times F(q)^{n-2}f(q) \\ \times[F(\overline{v})-F(q)]\,[1-F(\overline{v})] \end{array}\right]dq \\ +\left[\begin{array}{c} \frac{\partial u(B(\overline{v},r)+r)}{\partial r}nF(r)^{n-1} \\ \times[F(\overline{v})-F(r)]\,[1-F(\overline{v})] \end{array}\right] \\ +\frac{\partial u(B(\overline{v},r))}{\partial r}F(r)^{n}\,[1-F(\overline{v})] \end{array}\right\} \\ \\ +\frac{\partial u(B(\overline{v},r)+r)}{\partial r}nF(r)^{n-1}\,[F(\overline{v})-F(r)]\,[1-F(\overline{v})] \\ +\int\limits_{r}^{\overline{v}}\frac{\partial u(z+r)}{\partial r}nF(r)^{n-1}f(z)\,[F(\overline{v})-F(z)]\,dz \\ +\frac{\partial u(r)}{\partial r}F(r)^{n}\,[F(\overline{v})-F(r)] \end{array}\right\}.$$

As can be seen from Figure 5.3, there exists a unique $r^*(\overline{v})$ for any $\overline{v}$ such that $\frac{\partial U(\overline{v},r)}{\partial r}\geq 0$ for all $r\leq r^*(\overline{v})$ and $\frac{\partial U(\overline{v},r)}{\partial r}<0$ for all $r>r^*(\overline{v})$.[26] Thus, given the distribution of bidder valuations and the equilibrium value of $\overline{v}$, there exists a unique reserve price below which the seller's expected utility is increasing in $r$ and above which his expected utility is decreasing whilst the specific value of $r^*(\overline{v})$ ceteris paribus maximizes his expected utility.

[26] This in fact holds for any given values of $n$, $v_L$, $v_H$ and $\overline{v}$.

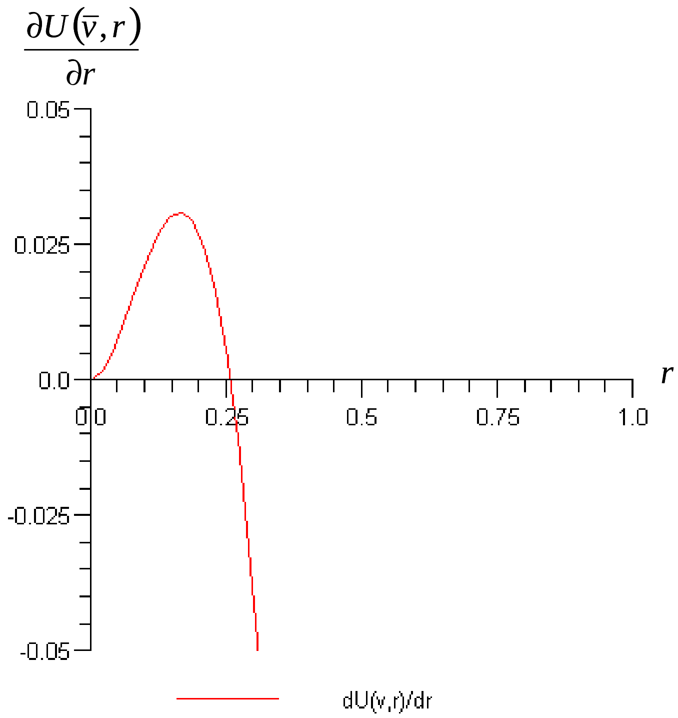


Figure 5.3: $\frac{\partial U(\overline{v},r)}{\partial r}$ for $n = 3$, $v_H = 1$, $v_L = 0$ and $\overline{v} = 0.5$

If the seller would not offer a Buyout Option but instead sell the two goods in a standard multi unit Vickrey auction with discriminatory pricing, as preliminarily described, his expected utility is:[27]

$$U_{SVA}(r) = \int_{r}^{v_H} \left\{ \begin{array}{l} \int_{r}^{y} \left\{ \begin{array}{l} \int_{r}^{z} u(z+q)\, l(q)\, dq \\ + \int_{v_L}^{r} u(z+r)\, l(q)\, dq \end{array} \right\} k(z)\, dz \\ \\ + \int_{v_L}^{r} u(r)\, k(z)\, dz \end{array} \right\} h(y)\, dy.$$

Again, denote by $y$ the largest of the $n+1$ realized $v_i$ and by $z$ and $q$ the second and third largest of the $n+1$ realized $v_i$ given $y$ and $z$ respectively. Further, $h(y)$, $k(z)$ and $l(q)$ define the probability density functions of $y$, $z$ and $q$. Clearly, the seller will only receive a positive expected utility if at least the highest bidder's valuation

[27] The seller's expected utility in a standard multi unit Vickrey auction with discriminatory pricing can easily be derived from the seller's expected utility function from the auction with a Buyout Price by simply eliminating the expected utility he could gain from offering the goods at the Buyout Price and exchanging the delimiters of the integers for the ones effective in the standard auction mechanism.

is drawn from $[r, v_H]$. If the highest bidder's valuation is instead below $r$, the seller would receive a utility of zero. If the highest bidder's valuation is between $r$ and $v_H$, his expected utility is his utility from either $z+q$ if both the second and third highest of the $n+1$ realized values of $v_i$ exceed the reserve price or from $z+r$ if only $z$ is above the reserve price. If however only the highest bidder's valuation would exceed or be equal to the reserve price, the seller's expected utility is $u(r)$ since only one good would be sold. If not a single bidder had a valuation above the reserve price, both goods remain unsold and the seller's utility is zero. The seller's expected utility from a standard multi unit Vickrey auction without a Buyout Option can therefore be summarized as follows:

$$E\left[U_{SVA}(r)\right] = \begin{cases} u(z+q) & \text{if } y \in [r, v_H],\ z \in [r, y] \text{ and } q \in [r, z] \\ u(z+r) & \text{if } y \in [r, v_H],\ z \in [r, y] \text{ and } q \in [v_L, r) \\ u(r) & \text{if } y \in [r, v_H] \text{ and } z \in [v_L, r) \\ 0 & \text{if } y \in [v_L, r) \end{cases}$$

Again, substituting for $h(y) = (n+1)F(y)^n f(y)$, $k(z) = \frac{nF(z)^{n-1}f(z)}{F(y)^n}$ and $l(q) = \frac{(n-1)F(q)^{n-2}f(q)}{F(z)^{n-1}}$ and for $F(v_L) = 0$ and $F(v_H) = 1$ yields:[28]

$$\begin{aligned} U_{SVA}(r) &= \int_r^{v_H} \left\{ \int_r^{z} \left[ \begin{array}{l} u(z+q)\, n(n+1) \\ \times (n-1) F(q)^{n-2} f(q) \end{array} \right] dq \right\} f(z)\left[1-F(z)\right] dz \\ &\quad + \int_r^{v_H} u(z+r)\, n(n+1) F(r)^{n-1} f(z)\left[1-F(z)\right] dz \\ &\quad + u(r)(n+1)F(r)^n \left[1-F(r)\right]. \end{aligned}$$

[28] See Appendix $A.7$.

In a standard multi unit Vickrey auction with discriminatory pricing, the seller would maximize his expected utility by choosing a reserve price $r_\alpha$ that fulfils the condition $\frac{\partial U_{SVA}(r_\alpha)}{\partial r_\alpha} = 0$. Therefore, the following condition must hold:

$$\begin{aligned}\frac{\partial U_{SVA}(r_\alpha)}{\partial r_\alpha} &= \int_{r_\alpha}^{v_H} \frac{\partial u(z+r_\alpha)}{\partial r_\alpha} n(n+1) F(r_\alpha)^{n-1} f(z)[1-F(z)]\,dz \\ &\quad -u(2r_\alpha)\, n(n+1) F(r_\alpha)^{n-1} f(r_\alpha)[1-F(r_\alpha)] \\ &\quad +\frac{\partial u(r_\alpha)}{\partial r_\alpha}(n+1) F(r_\alpha)^{n}[1-F(r_\alpha)] \\ &\quad +u(r_\alpha)(n+1) F(r_\alpha)^{n-1} f(r_\alpha)\{n[1-F(r_\alpha)]-F(r_\alpha)\}.\end{aligned}$$

If $\left.\frac{\partial U(\overline{v},r)}{\partial r}\right|_{r=r_\alpha} < 0$, it will follow that a seller in the auction enhanced with a Buyout Option would optimally choose a reserve price strictly lower than the reserve price maximizing his expected utility in the standard auction format without a Buyout Option (if $\left.\frac{\partial U(\overline{v},r)}{\partial r}\right|_{r=r_\alpha} < 0$, he would optimally choose a reserve price lower than $r_\alpha$ as it has been shown above since in that case he could strictly increase his expected utility by lowering the reserve price to some $r^*(\overline{v}) \leq r < r_\alpha$).

When substituting the optimal reserve price from the equilibrium condition of $\frac{\partial U_{SVA}(r_\alpha)}{\partial r_\alpha} = 0$ into $\frac{\partial U(\overline{v},r)}{\partial r}$, it follows that:

$$\left.\frac{\partial U(\overline{v},r)}{\partial r}\right|_{r=r_\alpha} = -(n+1)\left\{\begin{array}{l} nF(r)^{n-1} f(r)\left(1-F(\overline{v})\right)\left\{\begin{array}{l} u(r)-u(2r) \\ -u(B(\overline{v},r)) \\ \\ +u(B(\overline{v},r)+r) \end{array}\right\} \\ \\ -\frac{\partial B(\overline{v},r)}{\partial r}\left\{\begin{array}{l} \left[\begin{array}{c} 2\frac{\partial u(2B(\overline{v},r))}{\partial r} \\ \times\int\limits_{\overline{v}}^{v_H}\left[\begin{array}{c} nF(z)^{n-1} f(z) \\ \times[1-F(z)] \end{array}\right] dz \end{array}\right] \\ \\ +\int\limits_{r}^{\overline{v}}\left[\begin{array}{l} \frac{\partial u(B(\overline{v},r)+q)}{\partial r} n(n-1) \\ \times F(q)^{n-2} f(q) \\ \times[F(\overline{v})-F(q)] \\ \times[1-F(\overline{v})] \end{array}\right] dq \\ \\ +\left[\begin{array}{l} \frac{\partial u(B(\overline{v},r)+r)}{\partial r} nF(r)^{n-1} \\ \times[F(\overline{v})-F(r)] \\ \times[1-F(\overline{v})] \end{array}\right] \\ \\ +\frac{\partial u(B(\overline{v},r))}{\partial r} F(r)^n [1-F(\overline{v})] \end{array}\right\} \\ \\ -\frac{\partial u(B(\overline{v},r)+r)}{\partial r} nF(r)^{n-1}[F(\overline{v})-F(r)][1-F(\overline{v})] \\ \\ -\int\limits_{r}^{\overline{v}} \frac{\partial u(z+r)}{\partial r} nF(r)^{n-1} f(z)[F(\overline{v})-F(z)]\, dz \\ \\ +\frac{\partial u(r)}{\partial r} F(r)^n (1-F(\overline{v})) \\ \\ +\int\limits_{r}^{v_H} \frac{\partial u(z+r)}{\partial r} nF(r)^{n-1} f(z)[1-F(z)]\, dz \end{array}\right\}.$$

Define

$$
\begin{aligned}
Z(\overline{v},r)\big|_{r=r_\alpha} &= nF(r)^{n-1} f(r)(1-F(\overline{v}))\left\{\begin{array}{l} u(r)-u(2r)-u(B(\overline{v},r)) \\ \\ +u(B(\overline{v},r)+r) \end{array}\right\} \\
&\quad -\frac{\partial B(\overline{v},r)}{\partial r}\left\{\begin{array}{l} 2\frac{\partial u(2B(\overline{v},r))}{\partial r}\int\limits_{\overline{v}}^{v_H} nF(z)^{n-1} f(z)[1-F(z)]\,dz \\ +\int\limits_{r}^{\overline{v}}\left[\begin{array}{l} \frac{\partial u(B(\overline{v},r)+q)}{\partial r} n(n-1)F(q)^{n-2} f(q) \\ \times[F(\overline{v})-F(q)][1-F(\overline{v})] \end{array}\right]dq \\ +\frac{\partial u(B(\overline{v},r)+r)}{\partial r} nF(r)^{n-1}[F(\overline{v})-F(r)][1-F(\overline{v})] \\ +\frac{\partial u(B(\overline{v},r))}{\partial r} F(r)^n[1-F(\overline{v})] \end{array}\right\} \\
&\quad -\frac{\partial u(B(\overline{v},r)+r)}{\partial r} nF(r)^{n-1}[F(\overline{v})-F(r)][1-F(\overline{v})] \\
&\quad -\int\limits_{r}^{\overline{v}}\frac{\partial u(z+r)}{\partial r} nF(r)^{n-1} f(z)[F(\overline{v})-F(z)]\,dz \\
&\quad +\frac{\partial u(r)}{\partial r}F(r)^n(1-F(\overline{v})) \\
&\quad +\int\limits_{r}^{v_H}\frac{\partial u(z+r)}{\partial r} nF(r)^{n-1} f(z)[1-F(z)]\,dz.
\end{aligned}
$$

Thus, if $Z(\overline{v},r)\big|_{r=r_\alpha} > 0$ it will immediately follow that $\left.\frac{\partial U(\overline{v},r)}{\partial r}\right|_{r=r_\alpha} < 0$.

From Figure 5.4 it can be seen that $Z(\overline{v},r)\big|_{r=r_\alpha}$ is indeed strictly positive for any given value of $r_\alpha < \overline{v} < v_H$.[29] Therefore, it has been shown that $\left.\frac{\partial U(\overline{v},r)}{\partial r}\right|_{r=r_\alpha} < 0$ for any given value of $r_\alpha < \overline{v} < v_H$.

It has hence been shown that the seller will optimally choose a Buyout Price strictly lower than the highest bidder's threshold Buyout Price ($\overline{B} < B(v_H,r)$) and at the same time a reserve price lower than in the standard multi unit Vickrey auction without a Buyout Option ($r < r_\alpha$).

[29] $Z(\overline{v},r)\big|_{r=r_\alpha} > 0$ in fact holds for any given values of $n$, $v_L$ and $v_H$.

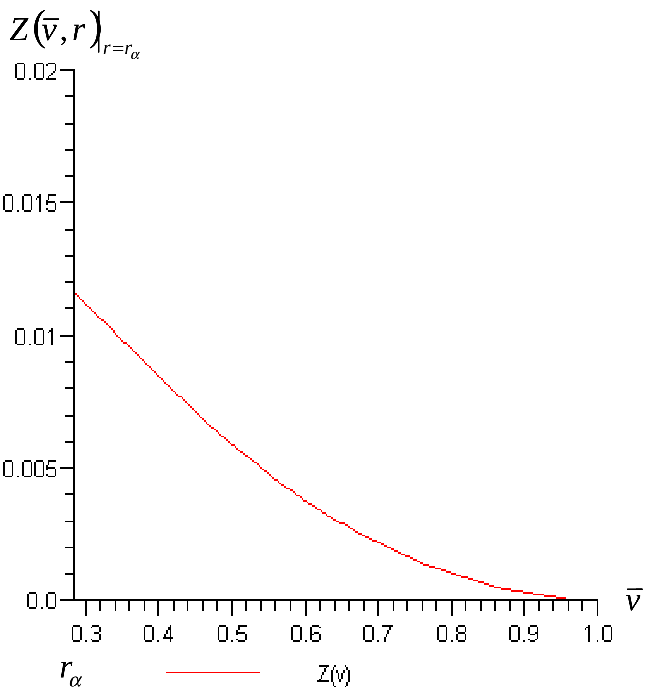


Figure 5.4: $Z(\bar{v}, r)|_{r=r_\alpha}$ for $n = 3$, $v_H = 1$, $v_L = 0$ and $r_\alpha$

# Chapter 6

# Discussion and Findings

Given that bidders follow the equilibrium strategy depicted above, allowing a seller to offer a Buyout Option increases his expected utility, if he optimally sets the reserve price and the Buyout Price. It is however not clear for which bidders the enhancement of the auction by a Buyout Option would indeed be beneficial. To address this issue, the bidders' welfare of the two auctions will hereafter be examined in detail. As it has been shown above, the seller will optimally choose a Buyout Price lower than the threshold Buyout Price for the bidder with the highest valuation ($\overline{B} < B(v_H, r)$) along with a reserve price lower than the reserve price maximizing his expected utility in a standard multi unit Vickrey auction ($r < r_\alpha$), given all bidders follow the equilibrium strategy $\widehat{S}(v)$. It will now be examined whether bidders have a higher expected payoff from participating in the auction enhanced with a Buyout Option or if they are better off in the standard multi unit Vickrey auction where the seller chooses a reserve price $r_\alpha$. In doing so, the expected individual bidder payoffs from the auction enhanced with a Buyout Option with $\overline{B} < B(v_H, r)$ and $r < r_\alpha$ will be compared with the respective expected individual bidder payoffs from the standard auction with no Buyout Option in place where the seller chooses $r = r_\alpha$.

Consider a bidder with valuation $v_i$ competing in a standard multi unit Vickrey auction without a Buyout Option where the seller sets $r = r_\alpha$. In this setting, such a bidder's expected payoff is given by:

$$\begin{aligned}\psi(v_i) &= \max\{v_i - r_\alpha, 0\} F(r_\alpha)^n \\ &+ \int_{r_\alpha}^{\max\{v_i, r_\alpha\}} (v_i - y) nF(y)^{n-1} f(y)\, dy \\ &+ \max\{v_i - r_\alpha, 0\} F(r_\alpha)^{n-1} (1 - F(v_i)) \\ &+ \int_{r_\alpha}^{\max\{v_i, r_\alpha\}} (v_i - y)(n-1)(1 - F(v_i)) F(y)^{n-2} f(y)\, dy.\end{aligned}$$

The first two terms of the sum are his expected payoff if $i$ indeed were the bidder with the highest valuation while the third and fourth terms are his expected payoff if he were the bidder with the second highest valuation. If bidder $i$ had a lower valuation, his expected payoff is zero.

$\psi(v_i)$ can be simplified and reduced to:[1]

$$\psi(v_i) = \int_{r_\alpha}^{\max\{v_i, r_\alpha\}} F(y)^n + (1 - F(v_i)) F(y)^{n-1}\, dy.$$

Next, consider an individual bidder's expected payoff when a Buyout Option is being offered. In the auction setup with a Buyout Option, as described in chapter 5, a bidder with $r \leq v_i \leq \overline{v}$ will choose not to exercise the Buyout Option but instead bid for one good in stage three of the game. His expected payoff therefore is:

$$\lambda(v_i) = \int_{r}^{v_i} F(y)^n + (1 - F(v_i)) F(y)^{n-1}\, dy.$$

[1] See Appendix $A.8$.

On the other hand, a bidder with $v_i > \overline{v}$ will exercise the Buyout Option in stage two and realize an expected payoff of

$$\gamma(v_i) = \left(v_i - \overline{B}(\overline{v}, r)\right) \frac{2\left(1 - F(\overline{v})^{n+1}\right) - (n+1)(1 - F(\overline{v}))F(\overline{v})^n}{(n+1)(1 - F(\overline{v}))}.$$

Substituting for $\overline{B}(\overline{v}, r) = \overline{v} - \left[\begin{array}{c} \frac{(n+1)(1-F(\overline{v}))}{2\left(1-F(\overline{v})^{n+1}\right)-(n+1)(1-F(\overline{v}))F(\overline{v})^n} \\ \times\left\{\int_r^{\overline{v}} F(y)^n + (1 - F(\overline{v}))F(y)^{n-1}\,dy\right\} \end{array}\right]$, $\gamma(v_i)$ can be rewritten as

$$\begin{aligned} \gamma(v_i) &= (v_i - \overline{v}) \frac{2\left(1 - F(\overline{v})^{n+1}\right) - (n+1)(1 - F(\overline{v}))F(\overline{v})^n}{(n+1)(1 - F(\overline{v}))} \\ &\quad + \int_r^{\overline{v}} F(y)^n + (1 - F(\overline{v}))F(y)^{n-1}\,dy. \end{aligned}$$

The expected payoffs from the two auction formats will now be examined. To start with, take any bidder with a valuation $v_i < r$. Such a bidder will receive an expected payoff of zero in both auction formats since he will not participate (in the case of $v_i = r$, the maximum payoff he could receive from successfully participating in the auction would be zero too). He is therefore indifferent with regard to his choice for either of the auction formats.

Consider now a bidder with $r \leq v_i \leq \overline{v}$. Such a bidder's expected payoff from participating in the auction with a Buyout Option is

$$\lambda(v_i) = \int_r^{v_i} F(y)^n + (1 - F(v_i))F(y)^{n-1}\,dy,$$

since he would not exercise the Buyout Option and only bid in stage three of the game. Such a bidder would only prefer the auction enhanced by a Buyout Option if $\lambda(v_i) > \psi(v_i)$.

Therefore, the following condition must hold for such a bidder to prefer the auction with a Buyout Option:

$$\int_{r}^{v_i} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy > \int_{r_\alpha}^{v_i} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy.$$

From $r < r_\alpha$ it follows that this condition clearly holds since the left hand side of the inequality strictly exceeds its right hand counterpart. Thus, a bidder with $r < v_i \leq \overline{v}$ has a strictly higher expected payoff if the seller were able to offer a Buyout Option, directly and solely originating form the fact that the seller would in that case choose a reserve price strictly lower than in the auction without a Buyout Option ($r < r_\alpha$).

To conclude, the expected payoffs for a bidder with $v_i > \overline{v}$ are being analyzed. Recall that such a bidder will always exercise the Buyout Option in stage two of the game since his expected payoff from exercising the Buyout Option strictly exceeds his expected payoff from not exercising the Buyout Option. Such a bidder would prefer an auction enhanced by a Buyout Option to the standard auction setting without a Buyout Option so long as his expected payoff from exercising the Buyout Option is higher than his expected payoff if he would participate in the auction without a Buyout Option. Define $\zeta(v_i, \overline{v}) = \gamma(v_i) - \psi(v_i)$. Therefore, $\zeta(v_i, \overline{v}) > 0$ must hold for such a bidder to be better off in the auction enhanced with a Buyout Option, or equivalently:

$$\begin{aligned}
\zeta(v_i, \overline{v}) &= (v_i - \overline{v}) \frac{2\left(1 - F(\overline{v})^{n+1}\right) - (n+1)(1 - F(\overline{v})) F(\overline{v})^n}{(n+1)(1 - F(\overline{v}))} \\
&\quad + \int_{r}^{\overline{v}} F(y)^n + (1 - F(\overline{v})) F(y)^{n-1} \, dy \\
&\quad - \int_{r_\alpha}^{v_i} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy \\
&> 0.
\end{aligned}$$

Recall that when the seller were able to offer a Buyout Option, he would choose $r < r_\alpha$.

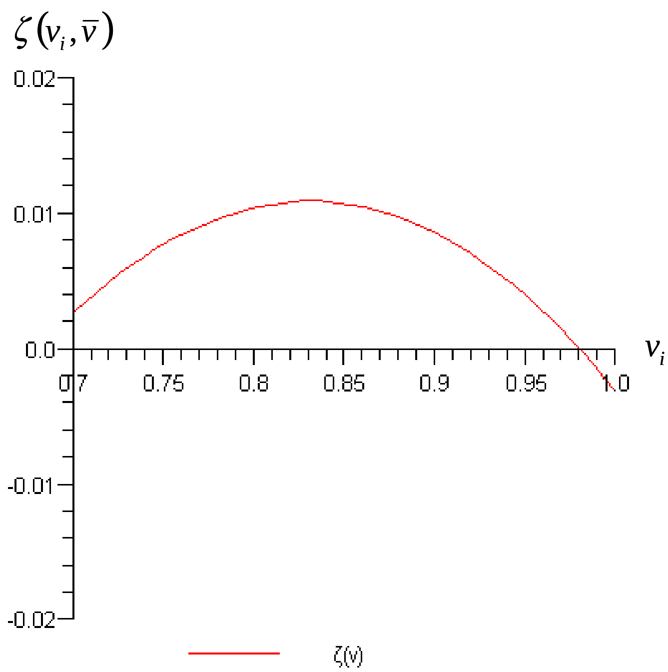


Figure 6.1: $\zeta(v_i, \overline{v})$ for $n = 3$, $v_H = 1$, $v_L = 0$, $\overline{v} = 0.7$, $r = 0.1$ and $r_\alpha$

From Figure 6.1 it can be seen that not for any bidder with $v_i > \overline{v}$, his expected payoff from the auction enhanced with a Buyout Option strictly exceeds his expected payoff from the standard multi unit Vickrey auction with no Buyout Option in place.[2] There exists a unique reference point $v^*(\overline{v})$ that determines which bidders would prefer the auction with a Buyout Option: Any bidder with $v_i < v^*(\overline{v})$ has a higher expected profit if the Buyout Option is in place while bidders with $v_i > v^*(\overline{v})$ would prefer the standard auction without a Buyout Option (that is, for $v_i < v^*(\overline{v})$ it would follow that $\zeta(v_i, \overline{v}) > 0$ while for $v_i > v^*(\overline{v})$, $\zeta(v_i, \overline{v})$ turns out to be negative).[3]

The welfare implications of all auction participants are summarized in the following Table 6.1. Since the seller will optimally choose a relatively low reserve price it turns out that not exercising the Buyout Option and only participating in the auction in stage three of the game is risky for some bidders. It therefore follows that bidders have relatively high incentives to exercise the Buyout Option in stage two of the game since the number of bidders indeed participating in the auction increases versus the standard auction without a Buyout Option. Moreover, since the seller chooses a Buyout Price lower than the highest bidder's valuation, the Buyout Option will be

[2] This in fact holds for any given values of $n$, $v_H$, $v_L$, $r$ and $\overline{v}$.
[3] Note however that a bidder with $v_i = v^*(\overline{v})$ again would be indifferent between the two auction formats.

executed with positive probability. If the Buyout Option is however being exercised by multiple bidders in stage two of the game, the bidders with the highest valuations for the goods will not receive them with certainty resulting in ex post inefficiency with positive probability.

Table 6.1: Model welfare comparison

| | | Expected Utility | | |
|---|---|---|---|---|
| | | SVA | Auction with Buyout Option | Comparison |
| Seller | | $U_{SVA}(r)$ | $U(\overline{v}, r)$ | $U(\overline{v}, r) > U_{SVA}(r)$ |
| Bidders | $v_i \leq r$ | 0 | 0 | $indifferent$ |
| | $r < v_i \leq \overline{v}$ | $\psi(v_i)$ | $\lambda(v_i)$ | $\lambda(v_i) > \psi(v_i)$ |
| | $\overline{v} < v_i \leq v^*(\overline{v})$ | $\psi(v_i)$ | $\gamma(v_i)$ | $\gamma(v_i) \geq \psi(v_i)$ |
| | $v^*(\overline{v}) < v_i$ | $\psi(v_i)$ | $\gamma(v_i)$ | $\gamma(v_i) < \psi(v_i)$ |

It has thereby been shown that an enhancement of the auction by a temporary Buyout Option need not lead to an increase in expected interim bidder welfare whatsoever. Bidders with relatively high valuations would prefer the auction without a Buyout Option (that is, bidders with $v_i > v^*(\overline{v})$). For all other bidders however the expected payoff from the auction with a Buyout Option exceeds their expected payoff from the auction where no Buyout Option is in place. The seller would at all times strictly prefer the auction with a Buyout Option. Thus, if the auction is enhanced by a Buyout Option, a pareto improvement may be achieved, but is not in all cases attainable.

# Chapter 7

# Conclusion

## 7.1 Summary

The aim of this thesis was to get a deeper understanding of the functioning of Buyout Options in simultaneous multi unit auctions and to highlight its relevance in real world auctions. The first introductory chapter outlined the topic of this thesis and stated its underlying research question and motivation.

Chapter 2 provided a brief overview on fundamental auction theory and its major findings. It constituted the two primary questions most relevant in the study of auction mechanisms, that is to say, their revenue implications and allocative efficiency. Clearly, it is generally in a seller's best interest to choose an auction format that allows him to gain the highest possible revenue whilst in view of the bidders, an auction that allocates the goods efficiently is optimal (needless to say that bidders likewise would opt for a mechanism that maximizes their payoff). Auctions are by their nature perceived as superior market institutions versus unpretentious fixed price markets since in their most conventional endowment, they allow for the possibility of both an increase in expected seller revenue and allocative efficiency in comparison to traditional fixed price institutions. The two most consequential findings and their key underlying assumptions for single unit auctions, i.e., the Revenue Equivalence Theorem and the Revenue Ranking Principle, were presented as well as a basic overview on the most common multi unit auction formats. Moreover, the problem of strategic manipulation in auctions was addressed and its potentially contrarian implications for both sides of the transaction were pictured.

Chapter 3 introduced the taxonomy and characteristics of Buyout Options in auctions. The specific properties of such options were illustrated on the basis of their particular duration of availability (permanent, temporary and limited Buyout Options) as well as in relation to the dynamics of their particular price level (static and dynamic Buyout Prices). It was shown that, from a seller's point of view, auctions enhanced with a Buyout Option may increase expected revenue since sellers can guarantee themselves a price for the good at sale that strictly exceeds the price that would have been met by a standard auction without such an option if bidders exercising the option would have indeed paid less in the standard auction. On the other hand however, when a Buyout Option is exercised by any of the bidders, the seller could run the risk of losing potential revenue if such bidders would have ended up paying more if a Buyout Option were not in place. Similarly, the decision on the execution of a Buyout Option or only traditionally bidding in the auction instead is a trade-off for the bidders since they could, on one hand, guarantee themselves a strictly positive payoff with positive probability when exercising the option but on the other hand bear the risk of losing gains they could have obtained when not exercising the option if the final price in the auction indeed would have been lower than the Buyout Price. Furthermore, the application and increasing significance of Buyout Options in present-day real world auctions was highlighted in a survey of distinguished online auction markets and a selection of auctions that have so far not been enhanced by such options was pictured.

The aim of chapter 4 was to provide a review of existing theoretical, empirical and experimental literature on Buyout Options in auctions. The topic has only very recently attracted the interest of economists, due to the fact that Buyout Options have merely been used in auctions since its inception on online auction platforms. According to the most commonly used auction format in practice, single unit ascending price auctions have hitherto predominantly and extensively been studied. At the outset of the theoretical literature on Buyout Options in auctions is the examination of the effects of auction participants' risk attitudes on the outcome of auctions.[1] The general findings are that when either the seller or the bidders are risk averse, a seller may increase his expected utility when offering a Buyout Option if he appropriately sets its price. Furthermore, even bidders may benefit from the availability of a Buyout Option if it allows them to strictly increase their expected utility. When bidders can indeed benefit from the execution of a Buyout Option, that is, if they are risk averse in some way, they are willing to pay a mark-up for the allocation of the good that is perceived as a premium for reducing the risk of losing the auction. Another alternative explanation for the use of Buyout Options in auctions

[1] See Budish and Takeyama (2001), Jung and Kim (2004), Reynolds and Wooders (2005), Chen et al. (2006), Hidvégi et al. (2006), Klumpp and Ranger (2006) and Mathews and Katzman (2006).

is if auction participants discount the value of future transactions.[2] In general, enhancing an auction with a Buyout Option when either side of the transaction is time impatient may reduce overall expected seller revenue or bidder payoffs but potentially allows the transaction to occur sooner and thereby increasing their respective utility. In addition to the considerations of risk aversion and time impatience, other transaction costs such as bidders' auction participation costs may explain the use and potential benefits of Buyout Options in auctions.[3] The availability of Buyout Options in auctions where bidders have substantial participation costs may increase both the seller's expected revenue and the bidders' expected payoffs by making the auction profitable for bidders who would not have participated in the auctions when a Buyout Option would not have been in place. Moreover, Buyout Options may be beneficial for sellers when intertemporally optimizing sequential auctions of multiple goods or taking into account the fact that homogeneous goods are being offered on alternative competing markets.[4] Besides these potentially beneficial implications of Buyout Options in auctions, its existence gives however rise to the possibility of inefficient auction outcomes since the goods may be allocated to bidders that do not value them most when bidders with the highest valuations miss out versus standard auctions without Buyout Options. Furthermore, the Revenue Equivalence Theorem may no longer hold when a Buyout Option is being offered. A pareto improvement is possible when enhancing an auction with a Buyout Option, it though critically depends on the level of the Buyout Price chosen by the seller. In conclusion, chapter 4 summarized existing empirical and experimental literature on such options in auctions. The fact that the analysis and empirical insights on Buyout Options in auctions gained to date are still relatively scarce ultimately makes its examination a "hot topic" in auction theory.

Chapter 5 finally contributed to the existing literature by examining a simultaneous multi unit Vickrey auction with discriminatory pricing enhanced with a temporary Buyout Option where a risk averse seller offers two homogeneous goods to an exogenously given number of bidders with single unit demand. To the best of my knowledge, it is the first attempt to analyze the impact of Buyout Options in simultaneous multi unit auctions, thereby making it a potentially valuable contribution to auction theory in general and to the literature on Buyout Options in auctions in particular. In a first step, optimal equilibrium bidder behaviour was derived that explicitly specifies under which circumstances bidders will exercise the Buyout Option

[2] See Mathews (2004), Mathews (2006) and Gupta and Gallien (2007).
[3] See Wang et al. (2004).
[4] See Lopomo (1998), Lee and Ahn (2004), Bose and Daripa (2006) and Kirkegaard and Overgaard (2007).

and when bidders would choose to only participate in the actual auction instead. A threshold Buyout Price increasing in both an individual bidder's valuation and the auction's reserve price for any arbitrary bidder was derived that characterizes a unique symmetric equilibrium. Subsequently, the analysis of optimal equilibrium seller behaviour revealed that he would maximize expected utility by choosing a Buyout Price lower than the highest bidder's threshold Buyout Price along with a reserve price lower than he would optimally choose in a standard auction without such a Buyout Option.

Chapter 6 presented the findings of the model analyzed in the antecedent chapter. With regard to the welfare implications of temporary Buyout Options in the proposed framework, a risk averse seller will always prefer to enhance the auction with such an option since his expected utility strictly exceeds the utility he could expect if he would choose to sell the goods by an auction without a Buyout Option. Bidders to a large extent would just as well benefit from the enhancement of the auction with a temporary Buyout Option. However, it has been shown that bidders with relatively high valuations would prefer the auction without a Buyout Option. Thus, a pareto improvement may be achieved when enhancing the auction with a Buyout Option but cannot be obtained in any case. Moreover, the adoption of a Buyout Option may introduce an ex post inefficient outcome of the auction since the goods are not awarded to the bidders who value them most with certainty - an objective that would be achieved when selling the goods by the standard auction instead. The analysis of the model has therefore shown that similar to single unit auctions, Buyout Options in simultaneous multi unit auctions may increase seller's expected utility and at the same time be beneficial to the bidders, ultimately endorsing its advantageous use for a wider range of auctions in practice.

## 7.2 Limitations and Future Research

A model has been formulated that allows for the analysis of a simultaneous multi unit auction enhanced with a temporary Buyout Option, motivated by the recent surge of literature on such options in auctions. Equilibrium strategies for a risk averse seller and risk neutral bidders have been characterized, using an independent private values framework. Despite its contribution to the understanding of both the functioning of Buyout Options in simultaneous multi unit auctions and its implications on the outcome of such auctions, the model discussed in this thesis is subject to several limitations. Due to the limitations of the model in this study, the conclusions that can be drawn from its results need to be regarded as preliminary in the discussion of the analysis of Buyout Options in simultaneous multi unit auctions, a topic that should be further examined, both analytically and empirically.

To start with, the assumption of independent private bidder valuations may not seem natural for numerous real world multi unit auctions. For instance, wherever the possibility of an ex post sale of the acquired goods from the auction or a correlation of the individual valuations is conceded, the bidder valuations obviously have some interdependency since a bidder's payoff not solely depends on his private valuation for the goods but at the same time critically depends on the subsequent resale price that can be obtained from selling the goods at some later point in time or the competing bidders' valuations. It could therefore be valuable to consider a common values or interdependent values setting since numerous goods on auction markets indeed have such components. Further to that, the seller could use a Buyout Price to post a signal on the true value of the good that may in turn ultimately result in higher expected seller revenue.

Additionally, more general preferences for the risk averse seller as well as the distribution of bidder valuations could be introduced to allow for more universal predictions on the implications of such Buyout Options in simultaneous multi unit auctions. It would be interesting if the results presented in the analysis above remain true in such a more general setting with an arbitrary seller utility function and any random distribution of bidder valuations.

The findings presented in this thesis are nevertheless quite general despite the fact that only the case for the simultaneous sale of two homogeneous goods with single unit bidder demand has been analyzed. Clearly, the natural extension of this work is to consider a model in which bidders feature multi unit demand and to augment the number of goods sold. The consideration of a model with multi unit demand would though add substantial notational and mathematical complexity to the analysis since the bidders' optimal equilibrium behaviour will critically depend on their marginal valuations for any additional good. Hence, when bidders feature multi unit demand, their optimal equilibrium bidding behaviour may considerably change depending on their respective marginal valuation, as it has been noted in chapter 2. However, even for this case Buyout Options may possibly be beneficial for auction participants given that such options can again guarantee them a strict increase in expected utility with positive probability. With regard to an increase in the number of goods offered in such an auction, it can be expected that, ceteris paribus, the results will not materially change since the major modification will be in respect of the probabilities with which bidders will receive a good upon exercising the Buyout Option or participating in the auction instead and their likelihood of indeed buying a good early at the ex ante prespecified fixed price. Albeit bidders will in that case most likely have lower threshold Buyout Prices, a risk averse seller will most likely choose a Buyout Price low enough to be exercised by some of the bidders with positive probability if it increases his expected utility. Thus, from the seller's perspective, he would as likely as not prefer to enhance such an auction with a Buyout Option since he may still gain higher expected utility when optimally choosing the Buyout Price.

The examination of entry costs to such auctions may also substantially affect equilibrium bidder behaviour, possibly resulting in alternative outcomes. Such additional costs imposed on potential buyers of the goods may on one hand deter them from participating in the auction since their incentives to actively bid are clearly diminished as their expectations of positive profits deteriorates with such costs. On the other hand however, Buyout Options could still guarantee these bidders a positive expected payoff at an ex ante predetermined fixed price and therefore potentially attract more bidders to participate, evidently resulting in a possible increase in expected seller revenue.

It is not trivial to explicitly determine the optimal Buyout Price. The analysis given here does not claim to offer a recipe for an explicit numerical prediction for the optimal Buyout Price but has instead pointed to the fact that there exists a well-defined range for the price to be potentially beneficial both for the seller and the bidders. An unambiguous prediction allowing a seller to choose a Buyout Price that maximizes his expected utility is only conceivable when additional assumptions would be made which in turn may alleviate the plausibility of the model.

Since only a single type of auction has been examined in the model presented, an examination of how the various types of alternative auction mechanisms for the simultaneous sale of multiple goods may impact the outcome of an auction would be enriching. Moreover, the study of the impact of Buyout Options on strategic manipulation in auctions would be interesting.

Considering the numerous limitations of the model and the diverse range of possible extensions, there still exists substantial need for future research on the topic of Buyout Options in simultaneous multi unit auctions. In conclusion, a continuation of this study, both analytically and empirically, is found to be a fruitful area for auction theory as well as practice.

# Appendix A

# Derivations for Chapters 5 and 6

## A.1 Derivation of $\lambda(v_i)$

Following the rule of partial integration

$$\int_a^b f'(x)\, g(x)\, dx = f(x)\, g(x)|_a^b - \int_a^b f(x)\, g'(x)\, dx,$$

the bidder's expected payoff function $\lambda(v_i)$ can be simplified by substituting for

$$\begin{aligned} & \int_r^{\min\{v_i,\overline{v}\}} (v_i - y)\, nF(y)^{n-1} f(y)\, dy \\ = \;& (v_i - \min\{v_i,\overline{v}\})\, F(\min\{v_i,\overline{v}\})^n - (v_i - r)\, F(r)^n + \int_r^{\min\{v_i,\overline{v}\}} F(y)^n\, dy \end{aligned}$$

and

$$\begin{aligned} & \int_r^{\min\{v_i,\overline{v}\}} (v_i - y)(n-1)(1 - F(v_i))\, F(y)^{n-2} f(y)\, dy \\ = \;& (v_i - \min\{v_i,\overline{v}\})(1 - F(v_i))\, F(\min\{v_i,\overline{v}\})^{n-1} \\ & - (v_i - r)(1 - F(v_i))\, F(r)^{n-1} + \int_r^{\min\{v_i,\overline{v}\}} (1 - F(v_i))\, F(y)^{n-1}\, dy. \end{aligned}$$

Therefore, it follows that

$$\begin{aligned}
\lambda(v_i) &= (v_i - r) F(r)^n + (v_i - \min\{v_i, \overline{v}\}) F(\min\{v_i, \overline{v}\})^n \\
&\quad - (v_i - r) F(r)^n + \int\limits_r^{\min\{v_i,\overline{v}\}} F(y)^n \, dy + (v_i - r) F(r)^{n-1} (1 - F(v_i)) \\
&\quad + (v_i - \min\{v_i, \overline{v}\}) (1 - F(v_i)) F(\min\{v_i, \overline{v}\})^{n-1} \\
&\quad - (v_i - r)(1 - F(v_i)) F(r)^{n-1} + \int\limits_r^{\min\{v_i,\overline{v}\}} (1 - F(v_i)) F(y)^{n-1} \, dy,
\end{aligned}$$

which can further be simplified to

$$\begin{aligned}
\lambda(v_i) &= (v_i - \min\{v_i, \overline{v}\}) \left[F(\min\{v_i, \overline{v}\})^n + (1 - F(v_i)) F(\min\{v_i, \overline{v}\})^{n-1}\right] \\
&\quad + \int\limits_r^{\min\{v_i,\overline{v}\}} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy.
\end{aligned}$$

## A.2 Derivation of $\gamma(v_i)$

Solving for $\binom{n}{j}$, the term $\sum\limits_{j=0}^{n} \frac{(1-F(\overline{v}))^j F(\overline{v})^{n-j}}{1+j} \binom{n}{j}$ can be extended to

$$\sum_{j=0}^{n} \frac{(1-F(\overline{v}))^j F(\overline{v})^{n-j}\, n!}{(1+j)\, j!\, (n-j)!}.$$

Since $(1+j)\, j! = (j+1)!$, the term can be restated as

$$\sum_{j=0}^{n} \frac{(1-F(\overline{v}))^j F(\overline{v})^{n-j}\, n!}{(j+1)!\, (n-j)!}$$

which can further be simplified by applying Newton's Binomial Formula, as follows:

According to Newton's Binomial Formula,

$$(a+b)^n = \sum_{j=0}^{n} a^{n-j} b^j \binom{n}{j} = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + ... + b^n,$$

the following relation can be deducted

$$\begin{aligned} (a+b)^{n+1} &= \sum_{j=0}^{n+1} a^{n+1-j} b^j \binom{n+1}{j} = a^{n+1} + \binom{n+1}{1} a^n b \\ &\quad + \binom{n+1}{2} a^{n-1} b^2 + \binom{n+1}{3} a^{n-2} b^3 + ... + b^{n+1} \end{aligned}$$

from which it follows that $\sum\limits_{j=0}^{n} \frac{a^{n-j} b^j n!}{(j+1)!(n-j)!}$ can be restated as $\frac{(a+b)^{n+1} - a^{n+1}}{(n+1)b}$.

Substituting $F(\overline{v})$ for $a$ and $(1-F(\overline{v}))$ for $b$ yields

$$\begin{aligned} \sum_{j=0}^{n} \frac{(1-F(\overline{v}))^j F(\overline{v})^{n-j}\, n!}{(j+1)!\, (n-j)!} &= \frac{(F(\overline{v}) + 1 - F(\overline{v}))^{n+1} - F(\overline{v})^{n+1}}{(n+1)(1-F(\overline{v}))} \\ &= \frac{1 - F(\overline{v})^{n+1}}{(n+1)(1-F(\overline{v}))}. \end{aligned}$$

Therefore, it follows that

$$\gamma\left(v_i\right)=\left(v_i-\overline{B}\right)\left[2\frac{1-F\left(\overline{v}\right)^{n+1}}{\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)}-F\left(\overline{v}\right)^n\right]$$

or

$$\gamma\left(v_i\right)=\left(v_i-\overline{B}\right)\frac{2\left(1-F\left(\overline{v}\right)^{n+1}\right)-\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^n}{\left(n+1\right)\left(1-F\left(\overline{v}\right)\right)}.$$

## A.3 Derivation of $\frac{\partial\lambda(v_i)}{\partial v_i}$

To generically determine bidder's marginal utility from not exercising the Buyout Option, a case differentiation for the two cases where $v_i < \overline{v}$ and $v_i > \overline{v}$ is necessary and the expected marginal payoff for either case must be separately analyzed.

First, consider bidder's marginal utility from not exercising the Buyout Option for all $v_i < \overline{v}$:

From $\lambda(v_i)$ it follows that

$$\lambda(v_i)|_{v_i<\overline{v}} = \int_r^{v_i} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy.$$

Differentiating over $v_i$ yields

$$\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}} = F(v_i)^n + (1 - F(v_i)) F(v_i)^{n-1} - \int_r^{v_i} f(v_i) F(y)^{n-1} \, dy.$$

Next consider marginal utility from not exercising the Buyout Option for all bidders with $v_i > \overline{v}$:

Again, from $\lambda(v_i)$ it follows that

$$\begin{aligned} \lambda(v_i)|_{v_i>\overline{v}} &= (v_i - \overline{v}) \left\{F(\overline{v})^n + (1 - F(v_i)) F(\overline{v})^{n-1}\right\} \\ &\quad + \int_r^{\overline{v}} F(y)^n + (1 - F(v_i)) F(y)^{n-1} \, dy. \end{aligned}$$

Differentiation over $v_i$ here yields

$$\begin{aligned} \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i>\overline{v}} &= F(\overline{v})^n + (1 - F(v_i)) F(\overline{v})^{n-1} \\ &\quad - (v_i - \overline{v}) F(\overline{v})^{n-1} f(v_i) - \int_r^{\overline{v}} f(v_i) F(y)^{n-1} \, dy. \end{aligned}$$

Therefore, the marginal expected payoff for any bidder with $v_i \neq \overline{v}$ can be restated as

$$\begin{aligned}\frac{\partial \lambda (v_i)}{\partial v_i} &= F(\min\{v_i, \overline{v}\})^n + (1 - F(v_i))\, F(\min\{v_i, \overline{v}\})^{n-1} \\ &\quad - (v_i - \min\{v_i, \overline{v}\})\, F(\min\{v_i, \overline{v}\})^{n-1} f(v_i) - \int\limits_r^{\min\{v_i, \overline{v}\}} f(v_i)\, F(y)^{n-1}\, dy.\end{aligned}$$

## A.4 Derivation of Probability Density Functions

To derive the distribution functions and probability density functions for the model at hand, order statistics need to be applied. Only the details necessary are here being presented to be able to relate to the methods used for the computation of the results.[1]

Assume $X_1, X_2, ...X_n$ are $n$ random variables independently drawn from a common continuous distribution function $F(x)$ with a probability density function $f(x)$. The associated order statistics are obtained by sorting the $n$ $x_i$'s in increasing order and denoted by $X_{1:n}, X_{2:n}, ..., X_{n:n}$, where $X_{n:n}$ is the highest order statistic denoting the largest of the $x_i$'s.

The cumulative distribution functions for any $X_{i:n}$ $(1 \leq i \leq n)$ are given by

$$F_{i:n}(x) = \Pr(X_{i:n} < x) = \sum_{r=i}^{n} \binom{n}{r} F(x)^r (1 - F(x))^{n-r} \tag{A.1}$$

and the probability density function for $X_{i:n}$ $(1 \leq i \leq n)$ by

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!} F(x)^{i-1} (1 - F(x))^{n-i} f(x). \tag{A.2}$$

If the population of variables is absolutely continuous, the joint probability density function of $X_{i:n}$ and $X_{j:n}$ (where $1 \leq i < j \leq n$) is

$$\begin{aligned} f_{i,j:n}(x_i, x_j) &= \frac{n!}{(i-1)!(j-i-1)!(n-j)!} F(x_i)^{i-1} \qquad \text{(A.3)} \\ &\quad \times (F(x_j) - F(x_i))^{j-i-1} (1 - F(x_j))^{n-j} f(x_i) f(x_j). \end{aligned}$$

[1] See Balakrishnan and Gupta (1998) and Balakrishnan and Rao (1998).

The joint probability density function of three or more variables $X_{i_1:n}, X_{i_2:n}, ..., X_{i_k:n}$ (with $1 \leq i_1 < i_2 < ... < i_k \leq n$) can be obtained from

$$\begin{aligned} f_{i_1,i_2,...,i_k:n}\left(x_{i_1}, x_{i_2}, ..., x_{i_k}\right) &= \frac{n!}{\left(i_1-1\right)!\left(i_2-i_1-1\right)!...\left(n-i_k\right)!} F\left(x_{i_1}\right)^{i_1-1} \quad \text{(A.4)} \\ &\times \left(F\left(x_{i_2}\right)-F\left(x_{i_1}\right)\right)^{i_2-i_1-1} \times ... \\ &\times \left(1-F\left(x_{i_k}\right)\right)^{n-i_k} f\left(x_{i_1}\right) f\left(x_{i_2}\right) ... f\left(x_{i_k}\right). \end{aligned}$$

Since the private valuation $v_i$ for any bidder $i$ in the model is a random variable drawn from $F(v)$, it follows that the corresponding order statistics can be derived by ordering the $n+1$ valuations in increasing order ($v_1$, $v_2$, ..., $v_{n+1}$, where $v_1$ denotes the smallest and $v_{n+1}$ the largest of these variables). Denote by $V_{1:n+1}$, $V_{2:n+1}$, ..., $V_{n+1:n+1}$ the order statistics where $V_{1:n+1}$ is the first order statistic denoting the smallest and $V_{n+1:n+1}$ is the highest order statistic (or $n+1^{\text{th}}$ order statistic) denoting the largest of the $v_i$'s. As for the model, the cumulative distribution function is given by $F(v)$ and the probability density function by $f(v) = \frac{\partial F(v)}{\partial v}$.

The cumulative distribution function of any order statistic $V_{i:n+1}$ $(1 \leq i \leq n+1)$ can then be derived from (A.1). Specifically, the cumulative distribution functions of $V_{n+1:n+1}$, $V_{n:n+1}$ and $V_{n-1:n+1}$ (the highest, second highest and third highest order statistic) are

$$\begin{aligned} F_{n+1:n+1}(v) &= \sum_{r=n+1}^{n+1} \binom{n+1}{r} F(v)^r \left(1-F(v)\right)^{n+1-r} \\ &= F(v)^{n+1}, \end{aligned}$$

$$\begin{aligned} F_{n:n+1}(v) &= \sum_{r=n}^{n+1} \binom{n+1}{r} F(v)^r \left(1-F(v)\right)^{n+1-r} \\ &= (n+1) F(v)^n \left(1-F(v)\right) + F(v)^{n+1} \end{aligned}$$

and

$$\begin{aligned}F_{n-1:n+1}(v) &= \sum_{r=n-1}^{n+1}\binom{n+1}{r}F(v)^r(1-F(v))^{n+1-r}\\ &= n(n+1)F(v)^{n-1}(1-F(v))^2\\ &\quad +(n+1)F(v)^n(1-F(v))+F(v)^{n+1}.\end{aligned}$$

Of specific interest for the model are the probability density functions to be able to compute the seller's expected utility. The probability density function for the largest of the $n+1$ realized $v_i$ (denoted $y$ in the model) can be obtained from (A.2):

$$f_y(y)=(n+1)F(y)^n f(y).$$

For the joint probability density function of $y$ and $z$, (A.3) can be applied:

$$f_z(z,y)=n(n+1)F(z)^{n-1}f(z)f(y),$$

and for the joint probability density function of $y$, $z$, and $q$, (A.4) is adopted, thus

$$f_q(q,y,z)=n(n+1)(n-1)F(q)^{n-2}f(q)f(z)f(y).$$

From here it follows that the probability density function of the largest of the $n+1$ realized $v_i$ is

$$h(y)=(n+1)F(y)^n f(y).$$

The probability density function of the second largest of the $n+1$ realized $v_i$ given the realized value of the largest of the $n+1$ realized $v_i$ is

$$\begin{aligned}k(z) &= k(z\mid y)=\frac{f_z(z,y)}{f_y(y)}=\frac{n(n+1)F(z)^{n-1}f(z)f(y)}{(n+1)F(y)^n f(y)}\\ &= \frac{nF(z)^{n-1}f(z)}{F(y)^n}.\end{aligned}$$

Accordingly, the probability density function of the third largest of the $n+1$ realized $v_i$ conditional on the largest and second largest realized $v_i$ is

$$\begin{aligned}
l(q) &= l(q \mid z, y) = \frac{f_q(q, y, z)}{f_z(z, y)} = \frac{n(n+1)(n-1)F(q)^{n-2} f(q) f(z) f(y)}{n(n+1)F(z)^{n-1} f(z) f(y)} \\
&= \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}}.
\end{aligned}$$

## A.5 Derivation of $U(\overline{v}, r)$

Substituting $h(y) = (n+1) F(y)^n f(y)$, $k(z) = \frac{nF(z)^{n-1} f(z)}{F(y)^n}$ and $l(q) = \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}}$ into $U(\overline{v}, r)$ yields:

$$
\begin{aligned}
U(\overline{v}, r) &= \int_{\overline{v}}^{v_H} \left\{ \begin{array}{l}
\int_{\overline{v}}^{y} u(2B(\overline{v}, r)) \frac{nF(z)^{n-1} f(z)}{F(y)^n} dz \\
+ \int_{r}^{\overline{v}} \left[ \begin{array}{l} \left\{ \begin{array}{l} \int_{r}^{z} \left[ \begin{array}{l} u(B(\overline{v}, r) + q) \\ \times \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}} \end{array} \right] dq \\ + \int_{v_L}^{r} \left[ \begin{array}{l} u(B(\overline{v}, r) + r) \\ \times \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}} \end{array} \right] dq \end{array} \right\} \\ \times \frac{nF(z)^{n-1} f(z)}{F(y)^n} \end{array} \right] dz \\
+ \int_{v_L}^{r} u(B(\overline{v}, r)) \frac{nF(z)^{n-1} f(z)}{F(y)^n} dz
\end{array} \right\} (n+1) F(y)^n f(y) \, dy \\
&+ \int_{r}^{\overline{v}} \left\{ \begin{array}{l}
\int_{r}^{y} \left[ \begin{array}{l} \left\{ \begin{array}{l} \int_{r}^{z} u(z+q) \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}} dq \\ + \int_{v_L}^{r} u(z+r) \frac{(n-1)F(q)^{n-2} f(q)}{F(z)^{n-1}} dq \end{array} \right\} \\ \times \frac{nF(z)^{n-1} f(z)}{F(y)^n} \end{array} \right] dz \\
+ \int_{v_L}^{r} u(r) \frac{nF(z)^{n-1} f(z)}{F(y)^n} dz
\end{array} \right\} (n+1) F(y)^n f(y) \, dy
\end{aligned}
$$

or

$$
\begin{aligned}
U\left(\overline{v},r\right) &= \int_{\overline{v}}^{v_H}\left\{\begin{array}{l}
\int_{\overline{v}}^{y} u\left(2B\left(\overline{v},r\right)\right)nF\left(z\right)^{n-1}f\left(z\right)dz \\
\\
+\int_{r}^{\overline{v}}\left\{\begin{array}{l}
\int_{r}^{z}\left[\begin{array}{l} u\left(B\left(\overline{v},r\right)+q\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\end{array}\right]dq \\
+\int_{v_L}^{r}\left[\begin{array}{l} u\left(B\left(\overline{v},r\right)+r\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\end{array}\right]dq
\end{array}\right\}nf\left(z\right)dz \\
\\
+\int_{v_L}^{r} u\left(B\left(\overline{v},r\right)\right)nF\left(z\right)^{n-1}f\left(z\right)dz
\end{array}\right\}\left(n+1\right)f\left(y\right)dy \\
&\quad +\int_{r}^{\overline{v}}\left\{\begin{array}{l}
\int_{r}^{y}\left\{\begin{array}{l}
\int_{r}^{z}\left[\begin{array}{l} u\left(z+q\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\end{array}\right]dq \\
+\int_{v_L}^{r}\left[\begin{array}{l} u\left(z+r\right)\left(n-1\right) \\ \times F\left(q\right)^{n-2}f\left(q\right)\end{array}\right]dq
\end{array}\right\}nf\left(z\right)dz \\
\\
+\int_{v_L}^{r} u\left(r\right)nF\left(z\right)^{n-1}f\left(z\right)dz
\end{array}\right\}\left(n+1\right)f\left(y\right)dy,
\end{aligned}
$$

which can further be simplified to

$$
\begin{aligned}
U(\overline{v},r) &= u(2B(\overline{v},r))\int_{\overline{v}}^{v_H} n(n+1)F(z)^{n-1}f(z)\left[F(v_H)-F(z)\right]dz \\
&+\int_r^{\overline{v}}\left[\begin{array}{l} u(B(\overline{v},r)+q)\,n(n+1)(n-1)F(q)^{n-2}f(q) \\ \times\left[F(\overline{v})-F(q)\right]\left[F(v_H)-F(\overline{v})\right] \end{array}\right]dq \\
&+\left[\begin{array}{l} u(B(\overline{v},r)+r)\,n(n+1)\left[F(r)^{n-1}-F(v_L)^{n-1}\right] \\ \times\left[F(\overline{v})-F(r)\right]\left[F(v_H)-F(\overline{v})\right] \end{array}\right] \\
&+u(B(\overline{v},r))\left[F(r)^n-F(v_L)^n\right](n+1)\left[F(v_H)-F(\overline{v})\right] \\
&+\int_r^{\overline{v}}\left\{\int_r^{z}\left[\begin{array}{l} u(z+q)\,n(n+1) \\ \times(n-1)F(q)^{n-2}f(q) \end{array}\right]dq\right\}f(z)\left[F(\overline{v})-F(z)\right]dz \\
&+\left[\begin{array}{l} n(n+1)\left[F(r)^{n-1}-F(v_L)^{n-1}\right] \\ \times\int_r^{\overline{v}}u(z+r)f(z)\left[F(\overline{v})-F(z)\right]dz \end{array}\right] \\
&+u(r)(n+1)\left[F(r)^n-F(v_L)^n\right]\left[F(\overline{v})-F(r)\right].
\end{aligned}
$$

Finally, substituting for $F(v_L)=0$ and $F(v_H)=1$ yields

$$
\begin{aligned}
U(\overline{v},r) &= u(2B(\overline{v},r))\int_{\overline{v}}^{v_H} n(n+1)F(z)^{n-1}f(z)\left[1-F(z)\right]dz \\
&+\int_r^{\overline{v}}\left[\begin{array}{l} u(B(\overline{v},r)+q)\,n(n+1)(n-1)F(q)^{n-2} \\ \times f(q)\left[F(\overline{v})-F(q)\right]\left[1-F(\overline{v})\right] \end{array}\right]dq \\
&+u(B(\overline{v},r)+r)\,n(n+1)F(r)^{n-1}\left[F(\overline{v})-F(r)\right]\left[1-F(\overline{v})\right] \\
&+u(B(\overline{v},r))F(r)^n(n+1)\left[1-F(\overline{v})\right] \\
&+\int_r^{\overline{v}}\left\{\int_r^{z}\left[\begin{array}{l} u(z+q)\,n(n+1) \\ \times(n-1)F(q)^{n-2}f(q) \end{array}\right]dq\right\}f(z)\left[F(\overline{v})-F(z)\right]dz \\
&+n(n+1)F(r)^{n-1}\int_r^{\overline{v}}u(z+r)f(z)\left[F(\overline{v})-F(z)\right]dz \\
&+u(r)(n+1)F(r)^n\left[F(\overline{v})-F(r)\right].
\end{aligned}
$$

## A.6 Derivation of $B(v_H, r)$

From

$$B(v,r) = v - \left[ \begin{array}{l} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left\{ \int\limits_r^v F(y)^n + (1-F(v))\, F(y)^{n-1}\, dy \right\} \end{array} \right]$$

and applying the rule of Bernoully-de l'Hosptial ($\lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$), thus

$$\begin{aligned} & \lim_{v\to v_H} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ = & \lim_{v\to v_H} \frac{-(n+1)f(v)}{-2(n+1)F(v)^n f(v) - (n+1)\left[-f(v)F(v)^n + (1-F(v))\, nF(v)^{n-1} f(v)\right]} \\ = & \lim_{v\to v_H} \frac{-1}{-F(v)^n - (1-F(v))\, nF(v)^{n-1}} \\ = & \; 1, \end{aligned}$$

it follows that $\lim\limits_{v\to v_H} B(v,r) = v_H - \int\limits_r^{v_H} F(y)^n\, dy$ and therefore

$$B(v_H, r) = v_H - \int\limits_r^{v_H} F(y)^n\, dy.$$

## A.7 Derivation of $U_{SVA}(r)$

When substituting $h(y) = (n+1)F(y)^n f(y)$, $k(z) = \frac{nF(z)^{n-1}f(z)}{F(y)^n}$ and $l(q) = \frac{(n-1)F(q)^{n-2}f(q)}{F(z)^{n-1}}$ into $U_{SVA}(r)$ it follows that

$$U_{SVA}(r) = \int_r^{v_H} \left[ \begin{array}{l} \left\{ \begin{array}{l} \int_r^y \left\{ \begin{array}{l} \int_r^z u(z+q) \frac{(n-1)F(q)^{n-2}f(q)}{F(z)^{n-1}} dq \\ +\int_{v_L}^r u(z+r) \frac{(n-1)F(q)^{n-2}f(q)}{F(z)^{n-1}} dq \end{array} \right\} \frac{nF(z)^{n-1}f(z)}{F(y)^n} dz \\ \\ +\int_{v_L}^r u(r) \frac{nF(z)^{n-1}f(z)}{F(y)^n} dz \end{array} \right\} \\ \\ \times (n+1)F(y)^n f(y) \end{array} \right] dy.$$

From rearranging it follows that

$$\begin{aligned} U_{SVA}(r) &= \int_r^{v_H} \left\{ \int_r^z \left[ \begin{array}{l} u(z+q)n(n+1) \\ \times (n-1)F(q)^{n-2} f(q) \end{array} \right] dq \right\} f(z)\left[F(v_H) - F(z)\right] dz \\ &+ \int_r^{v_H} \left[ \begin{array}{l} u(z+r)n(n+1)\left[F(r)^{n-1} - F(v_L)^{n-1}\right] \\ \times f(z)\left[F(v_H) - F(z)\right] \end{array} \right] dz \\ &+ u(r)(n+1)\left[F(r)^n - F(v_L)^n\right]\left[F(v_H) - F(r)\right]. \end{aligned}$$

Further simplifying by substituting for $F(v_L) = 0$ and $F(v_H) = 1$ yields

$$\begin{aligned} U_{SVA}(r) &= \int_r^{v_H} \left\{ \int_r^z \left[ \begin{array}{l} u(z+q)n(n+1) \\ \times (n-1)F(q)^{n-2} f(q) \end{array} \right] dq \right\} f(z)\left[1 - F(z)\right] dz \\ &+ \int_r^{v_H} u(z+r)n(n+1)F(r)^{n-1} f(z)\left[1 - F(z)\right] dz \\ &+ u(r)(n+1)F(r)^n\left[1 - F(r)\right]. \end{aligned}$$

## A.8 Derivation of $\psi(v_i)$

Substituting for

$$\begin{aligned}
&\int_{r_\alpha}^{\max\{v_i,r_\alpha\}} (v_i - y)\, nF(y)^{n-1} f(y)\, dy \\
= \;& (v_i - \max\{v_i, r_\alpha\})\, F(\max\{v_i, r_\alpha\})^n - (v_i - r_\alpha)\, F(r_\alpha)^n \\
&+ \int_{r_\alpha}^{\max\{v_i,r_\alpha\}} F(y)^n\, dy
\end{aligned}$$

and

$$\begin{aligned}
&\int_{r_\alpha}^{\max\{v_i,r_\alpha\}} (v_i - y)(n-1)(1 - F(v_i))\, F(y)^{n-2} f(y)\, dy \\
= \;& (v_i - \max\{v_i, r_\alpha\})\, F(\max\{v_i, r_\alpha\})^{n-1} (1 - F(v_i)) \\
& - (v_i - r_\alpha)\, F(r_\alpha)^{n-1} (1 - F(v_i)) + \int_{r_\alpha}^{\max\{v_i,r_\alpha\}} F(y)^{n-1} (1 - F(v_i))\, dy
\end{aligned}$$

into $\psi(v_i)$ yields

$$\begin{aligned}\psi(v_i) &= \max\{v_i - r_\alpha, 0\} F(r_\alpha)^n + (v_i - \max\{v_i, r_\alpha\}) F(\max\{v_i, r_\alpha\})^n \\ &\quad - (v_i - r_\alpha) F(r_\alpha)^n + \int\limits_{r_\alpha}^{\max\{v_i, r_\alpha\}} F(y)^n \, dy \\ &\quad + \max\{v_i - r_\alpha, 0\} \left[F(r_\alpha)^{n-1} (1 - F(v_i))\right] \\ &\quad + (v_i - \max\{v_i, r_\alpha\}) F(\max\{v_i, r_\alpha\})^{n-1} (1 - F(v_i)) \\ &\quad - (v_i - r_\alpha) F(r_\alpha)^{n-1} (1 - F(v_i)) \\ &\quad + \int\limits_{r_\alpha}^{\max\{v_i, r_\alpha\}} F(y)^{n-1} (1 - F(v_i)) \, dy,\end{aligned}$$

which can further be simplified to

$$\begin{aligned}\psi(v_i) &= \max\{v_i - r_\alpha, 0\} F(r_\alpha)^{n-1} [F(r_\alpha) + (1 - F(v_i))] \\ &\quad + (v_i - \max\{v_i, r_\alpha\}) F(\max\{v_i, r_\alpha\})^{n-1} [F(\max\{v_i, r_\alpha\}) + (1 - F(v_i))] \\ &\quad - (v_i - r_\alpha) F(r_\alpha)^{n-1} [F(r_\alpha) + (1 - F(v_i))] \\ &\quad + \int\limits_{r_\alpha}^{\max\{v_i, r_\alpha\}} F(y)^n + F(y)^{n-1} (1 - F(v_i)) \, dy.\end{aligned}$$

A case differentiation is needed for the two cases of $v_i > r_\alpha$ and $v_i < r_\alpha$:

For a bidder with $v_i > r_\alpha$, $\psi(v_i)$ is given by

$$\psi(v_i)\big|_{v_i > r_\alpha} = \int_{r_\alpha}^{v_i} F(y)^n + F(y)^{n-1}(1 - F(v_i))\, dy$$

and for a bidder with $v_i < r_\alpha$ by

$$\psi(v_i)\big|_{v_i < r_\alpha} = \int_{r_\alpha}^{r_\alpha} F(y)^n + F(y)^{n-1}(1 - F(v_i))\, dy.$$

Thus, $\psi(v_i)$ for either case can be stated as

$$\psi(v_i) = \int_{r_\alpha}^{\max\{v_i, r_\alpha\}} F(y)^n + (1 - F(v_i))\, F(y)^{n-1}\, dy.$$

# Appendix B

# Mathematical Proofs for Chapters 5 and 6

## B.1 Proof of $\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}} < \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$

**Proof.** Start by noting that

$$\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}} = F(v_i)^n + (1-F(v_i))\,F(v_i)^{n-1} - \int_r^{v_i} f(v_i)\,F(y)^{n-1}\,dy$$

and

$$\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} = F(\overline{v})^n + (1-F(\overline{v}))\,F(\overline{v})^{n-1} - \int_r^{\overline{v}} f(\overline{v})\,F(y)^{n-1}\,dy.$$

Define $\Theta(\overline{v},v_i) = \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} - \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}}$. Therefore it follows that

$$\begin{aligned}\Theta(\overline{v},v_i) &= F(\overline{v})^n - F(v_i)^n \\ &\quad + (1-F(\overline{v}))\,F(\overline{v})^{n-1} - (1-F(v_i))\,F(v_i)^{n-1} \\ &\quad - \int_r^{\overline{v}} f(\overline{v})\,F(y)^{n-1}\,dy + \int_r^{v_i} f(v_i)\,F(y)^{n-1}\,dy.\end{aligned}$$

If it can be shown that $\left.\Theta(\overline{v},v_i)\right|_{v_i=\overline{v}} \geq 0$ and $\frac{\partial\Theta(\overline{v},v_i)}{\partial v_i} < 0$, it will follow that $\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}} < \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$.

Start by noting that

$$\Theta\left(\overline{v}, v_i\right)\big|_{v_i=\overline{v}} = 0.$$

Furthermore,

$$\begin{aligned}\frac{\partial\Theta\left(\overline{v}, v_i\right)}{\partial v_i} &= -nF\left(v_i\right)^{n-1} f\left(v_i\right) + f\left(v_i\right) F\left(v_i\right)^{n-1} \\ &\quad - \left(1 - F\left(v_i\right)\right)\left(n-1\right) F\left(v_i\right)^{n-2} f\left(v_i\right) \\ &\quad + \int_r^{v_i} \frac{\partial f\left(v_i\right)}{\partial v_i} F\left(y\right)^{n-1} dy + f\left(v_i\right) F\left(v_i\right)^{n-1},\end{aligned}$$

which equals[1]

$$\frac{\partial\Theta\left(\overline{v}, v_i\right)}{\partial v_i} = -f\left(v_i\right) F\left(v_i\right)^{n-2}\left[\left(n-2\right) F\left(v_i\right) + \left(1 - F\left(v_i\right)\right)\left(n-1\right)\right] < 0.$$

It has thereby been proven that $\left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i<\overline{v}} < \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$. ■

[1] Since $\frac{\partial f(v)}{\partial v} = 0$ (due to the nature of the uniform distribution).

## B.2 Proof of $\left.\frac{\partial\gamma(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} > \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$

**Proof.** The condition $\left.\frac{\partial\gamma(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} > \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$ can be restated as

$$\begin{aligned}
&\frac{2\left(1-F\left(\overline{v}\right)^{n+1}\right)-(n+1)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n}}{(n+1)\left(1-F\left(\overline{v}\right)\right)} \\
&-\left\{F\left(\overline{v}\right)^{n}+\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n-1}-\int\limits_{r}^{\overline{v}} f\left(\overline{v}\right)F\left(y\right)^{n-1}dy\right\} \\
>\ & 0,
\end{aligned}$$

which equals

$$\begin{aligned}
&\frac{2\left(1-F\left(\overline{v}\right)^{n+1}\right)-(n+1)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n}}{(n+1)\left(1-F\left(\overline{v}\right)\right)} \\
&-F\left(\overline{v}\right)^{n-1}+\int\limits_{r}^{\overline{v}} f\left(\overline{v}\right)F\left(y\right)^{n-1}dy \\
>\ & 0
\end{aligned}$$

or

$$\begin{aligned}
&\frac{1}{(n+1)\left(1-F\left(\overline{v}\right)\right)}\left\{\begin{array}{l} 2\left(1-F\left(\overline{v}\right)^{n+1}\right)-(n+1)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n} \\ -(n+1)\left(1-F\left(\overline{v}\right)\right)F\left(\overline{v}\right)^{n-1} \end{array}\right\} \\
&+\int\limits_{r}^{\overline{v}} f\left(\overline{v}\right)F\left(y\right)^{n-1}dy \\
>\ & 0.
\end{aligned}$$

Define

$$a(\overline{v}) = \frac{1}{(n+1)(1-F(\overline{v}))},$$

$$\begin{aligned} b(\overline{v}) &= 2\left(1-F(\overline{v})^{n+1}\right)-(n+1)(1-F(\overline{v}))F(\overline{v})^{n} \\ &\quad -(n+1)(1-F(\overline{v}))F(\overline{v})^{n-1} \end{aligned}$$

and

$$c(\overline{v}) = \int_{r}^{\overline{v}} f(\overline{v})F(y)^{n-1}\,dy.$$

It remains to be shown that $a(\overline{v})$, $b(\overline{v})$, and $c(\overline{v})$ are positive for all $F(\overline{v})<1$.

$a(\overline{v})>0$ clearly holds since $1-F(\overline{v})>0$.

Next, consider $b(\overline{v})$:

Define

$$A(\overline{v}) = 2\left(1-F(\overline{v})^{n+1}\right)-(n+1)(1-F(\overline{v}))F(\overline{v})^{n}-(n+1)(1-F(\overline{v}))F(\overline{v})^{n-1}.$$

If $A(v_H)\geq 0$ and $\frac{\partial A(\overline{v})}{\overline{v}}<0$ for any $\overline{v}<v_H$, it will follow that $A(\overline{v})>0$ for all $F(\overline{v})<1$.

Since $F(v_H)=1$ it follows that

$$\begin{aligned} A(v_H) &= 2\left(1-F(v_H)^{n+1}\right)-(n+1)(1-F(v_H))F(v_H)^{n} \\ &\quad -(n+1)(1-F(v_H))F(v_H)^{n-1} \\ &= 0. \end{aligned}$$

Furthermore,

$$\begin{aligned}\frac{\partial A(\overline{v})}{\partial \overline{v}} &= -2(n+1)F(\overline{v})^n f(\overline{v}) - (n+1)\left\{-f(\overline{v})F(\overline{v})^n + (1-F(\overline{v}))nF(\overline{v})^{n-1}f(\overline{v})\right\} \\ &\quad -(n+1)\left\{-f(\overline{v})F(\overline{v})^{n-1} + (1-F(\overline{v}))(n-1)F(\overline{v})^{n-2}f(\overline{v})\right\}\end{aligned}$$

can be simplified to

$$\frac{\partial A(\overline{v})}{\partial \overline{v}} = -(n+1)(n-1)f(\overline{v})F(\overline{v})^{n-2}\left(1-F(\overline{v})^2\right),$$

which is negative for all $F(\overline{v}) < 1$. Therefore, it has been proven that $b(\overline{v}) > 0$.

$c(\overline{v}) > 0$ since $f(\overline{v}) > 0$ and $\int\limits_r^{\overline{v}} F(y)^{n-1}\,dy > 0$.

Since $a(\overline{v}) > 0$, $b(\overline{v}) > 0$ and $c(\overline{v}) > 0$ for all $F(\overline{v}) < 1$, it follows that $a(\overline{v}) \times b(\overline{v}) \times c(\overline{v}) > 0$ for all $F(\overline{v}) < 1$. It has thereby been proven that $\left.\frac{\partial\gamma(v_i)}{\partial v_i}\right|_{v_i=\overline{v}} > \left.\frac{\partial\lambda(v_i)}{\partial v_i}\right|_{v_i=\overline{v}}$ for all $F(\overline{v}) < 1$. ■

## B.3 Proof of $\frac{\partial B(v,r)}{\partial v} > 0$

**Proof.** The first derivative of the threshold Buyout Price subject to $v$ is

$$\frac{\partial B(v,r)}{\partial v} = 1 - \left\{ \begin{array}{l} - \left[ \begin{array}{l} \frac{(n+1)f(v)}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left\{ \int\limits_r^v F(y)^n + (1-F(v)) F(y)^{n-1} \, dy \right\} \end{array} \right] \\ \\ - \left[ \begin{array}{l} \frac{(n+1)(1-F(v)) \left[ \begin{array}{l} -2(n+1) F(v)^n f(v) \\ +(n+1) F(v)^n f(v) \\ -(n+1)(1-F(v)) nF(v)^{n-1} f(v) \end{array} \right]}{\left[2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n\right]^2} \\ \times \left\{ \int\limits_r^v F(y)^n + (1-F(v)) F(y)^{n-1} \, dy \right\} \end{array} \right] \\ \\ + \left[ \begin{array}{l} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left[ \begin{array}{l} -\int\limits_r^v f(v) F(y)^{n-1} \, dy + F(v)^n \\ +(1-F(v)) F(v)^{n-1} \end{array} \right] \end{array} \right] \end{array} \right\}.$$

Rearranging yields

$$\begin{aligned} \frac{\partial B(v,r)}{\partial v} &= 1 - \left[ \begin{array}{l} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left[ F(v)^{n-1} - \int\limits_r^v f(v) F(y)^{n-1} \, dy \right] \end{array} \right] \\ &\quad + \left[ \begin{array}{l} \frac{(n+1)f(v)}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \\ \times \left\{ 1 + \frac{(n+1)(1-F(v))\left[-2F(v)^n + F(v)^n - (1-F(v))nF(v)^{n-1}\right]}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^n} \right\} \\ \times \left\{ \int\limits_r^v F(y)^n + (1-F(v)) F(y)^{n-1} \, dy \right\} \end{array} \right]. \end{aligned}$$

Define

$$a(v) = 1 - \left[ \begin{array}{c} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} \\ \times \left[ F(v)^{n-1} - \int\limits_{r}^{v} f(v) F(y)^{n-1} dy \right] \end{array} \right]$$

and

$$b(v) = \left[ \begin{array}{l} \frac{(n+1)f(v)}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} \\ \times \left\{ 1 + \frac{(n+1)(1-F(v))\left[-2F(v)^{n}+F(v)^{n}-(1-F(v))nF(v)^{n-1}\right]}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} \right\} \\ \times \left\{ \int\limits_{r}^{v} F(y)^{n} + (1-F(v)) F(y)^{n-1} dy \right\} \end{array} \right].$$

For $\frac{\partial B(v,r)}{\partial v}$ to be positive it remains to be proven that $a(v) > 0$ and $b(v) > 0$.

It will first be shown that $a(v) > 0$:

$$a(v) = 1 - \left[ \begin{array}{c} \frac{(n+1)(1-F(v))}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} \\ \times \left[ F(v)^{n-1} - \int\limits_{r}^{v} f(v) F(y)^{n-1} dy \right] \end{array} \right]$$

is equal to

$$\begin{aligned} a(v) &= \frac{1}{2\left(1-F(v)^{n+1}\right) - (n+1)(1-F(v))F(v)^{n}} \\ &\quad \times \left\{ \begin{array}{l} 2\left(1-F(v)^{n+1}\right) - (n+1)(1-F(v))F(v)^{n} \\ -(n+1)(1-F(v))F(v)^{n-1} \\ +(n+1)(1-F(v)) \int\limits_{r}^{v} f(v) F(y)^{n-1} dy \end{array} \right\}. \end{aligned}$$

Note that $\frac{1}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} > 0$ since

$$\left(1-F\left(v\right)^{n+1}\right)-\left(n+1\right)\left(1-F\left(v\right)\right)F\left(v\right)^{n}$$

can be rewritten as

$$\begin{aligned}
& \left(1-F\left(v\right)\right)\left(1+F\left(v\right)+F\left(v\right)^{2}+...+F\left(v\right)^{n}\right)-\left(n+1\right)\left(1-F\left(v\right)\right)F\left(v\right)^{n} \\
= & \left(1-F\left(v\right)\right)\left[1+F\left(v\right)+F\left(v\right)^{2}+...+F\left(v\right)^{n}-\left(n+1\right)F\left(v\right)^{n}\right] \\
= & \left(1-F\left(v\right)\right)\left[\sum_{i=0}^{n}F\left(v\right)^{i}-\left(n+1\right)F\left(v\right)^{n}\right],
\end{aligned}$$

which is positive for all $F\left(v\right)<1$ since $\sum\limits_{i=0}^{n}F\left(v\right)^{i}>\left(n+1\right)F\left(v\right)^{n}>0$. From here it directly follows that $2\left(1-F\left(v\right)^{n+1}\right)-\left(n+1\right)\left(1-F\left(v\right)\right)F\left(v\right)^{n}>0$ for all $F\left(v\right)<1$. Further, note that $\left(n+1\right)\left(1-F\left(v\right)\right)\int\limits_{r}^{v}f\left(v\right)F\left(y\right)^{n-1}dy>0$ and that $2\left(1-F\left(v\right)^{n+1}\right)-\left(n+1\right)\left(1-F\left(v\right)\right)F\left(v\right)^{n}-\left(n+1\right)\left(1-F\left(v\right)\right)F\left(v\right)^{n-1}>0$.[2] It has therefore been proven that that $a\left(v\right)>0$ for all $F\left(v\right)<1$.

In a second step it is being shown that $b\left(v\right)>0$:

Recall that

$$b\left(v\right)=\left[\begin{array}{l}
\frac{(n+1)f(v)}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} \\
\times\left\{1+\frac{(n+1)(1-F(v))\left[-2F(v)^{n}+F(v)^{n}-(1-F(v))nF(v)^{n-1}\right]}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}}\right\} \\
\times\left\{\int\limits_{r}^{v}F\left(y\right)^{n}+\left(1-F\left(v\right)\right)F\left(y\right)^{n-1}dy\right\}
\end{array}\right].$$

[2] See Proof in Appendix $B$.2..

Define

$$c(v) = \frac{(n+1) f(v)}{2\left(1 - F(v)^{n+1}\right) - (n+1)(1 - F(v)) F(v)^n},$$

$$d(v) = 1 + \frac{(n+1)(1 - F(v)) \left[-2F(v)^n + F(v)^n - (1 - F(v)) nF(v)^{n-1}\right]}{2\left(1 - F(v)^{n+1}\right) - (n+1)(1 - F(v)) F(v)^n}$$

and

$$e(v) = \int_r^v F(y)^n + (1 - F(v)) F(y)^{n-1} \, dy.$$

As it has already been shown above, $c(v) > 0$ for all $F(v) < 1$. Clearly $e(v) > 0$ for all $F(v) < 1$. What remains to be shown is that $d(v) > 0$ for all $F(v) < 1$. Rearranging $d(v)$ yields

$$\begin{aligned} d(v) &= \frac{1}{2\left(1 - F(v)^{n+1}\right) - (n+1)(1 - F(v)) F(v)^n} \\ &\quad \times \left\{2\left(1 - F(v)^{n+1}\right) - (n+1)(1 - F(v)) F(v)^{n-1} \{2F(v) + (1 - F(v)) n\}\right\}. \end{aligned}$$

Define

$$g(v) = 2\left(1 - F(v)^{n+1}\right) - (n+1)(1 - F(v)) F(v)^{n-1} \{2F(v) + (1 - F(v)) n\}.$$

For $d(v)$ to be positive, it suffices to show that $g(v) > 0$. If $g(v_H) \geq 0$ and $\frac{\partial g(v)}{\partial v} < 0$ it is proven that $g(v) > 0$ for all $v < v_H$.

First, note that

$$g(v_H) = 0.$$

Furthermore,

$$\begin{aligned} \frac{\partial g(v)}{\partial v} &= -2(n+1) F(v)^n f(v) \\ &\quad - (n+1) f(v) F(v)^{n-2} \left\{ \begin{array}{l} \left[ \begin{array}{l} [-F(v) + (1 - F(v))(n-1)] \\ \times \{2F(v) + (1 - F(v)) n\} \end{array} \right] \\ + (1 - F(v)) F(v) [2 - n] \end{array} \right\} \end{aligned}$$

or by rearranging

$$\frac{\partial g(v)}{\partial v} = -n(n+1)(n-1)f(v)F(v)^{n-2}[1-F(v)]^2 < 0.$$

Thereby it has been proven that $b(v) > 0$ for all $F(v) < 1$.

Since $a(v) > 0$ and $b(v) > 0$ for all $F(v) < 1$ it follows that $\frac{\partial B(v,r)}{\partial v} > 0$ for all $F(v) < 1$. It has therefore been proven that $\frac{\partial B(v,r)}{\partial v} > 0$ for all $F(v) < 1$. $\blacksquare$

## B.4 Proof of $\frac{\partial B(v,r)}{\partial r} > 0$

**Proof.**

$\frac{\partial B(v,r)}{\partial r}$ clearly is positive for all $F(v) < 1$ since $\frac{(n+1)[1-F(v)]}{2\left(1-F(v)^{n+1}\right)-(n+1)(1-F(v))F(v)^{n}} > 0$ and $F(r)^n + (1-F(v))\,F(r)^{n-1} > 0$ for all $F(v) < 1$.[3] It has thereby been proven that $\frac{\partial B(v,r)}{\partial r} > 0$ for all $F(v) < 1$. ■

[3] See Appendix $B.3$.

# Bibliography


Ackerberg, D., K. Hirano, and Q. Shariar (2006): "The Buy-it-now Option, Risk Aversion, and Impatience in an Empirical Model of eBay Bidding," University of California Working Paper.

Anderson, S., D. Friedman, G. Milam, and N. Singh (2004): "Buy it Now: A Hybrid Internet Market Institution," University of California Working Paper.

Anwar, S., R. McMillan, and M. Zheng (2006): "Bidding behavior in competing auctions: Evidence from eBay," *European Economic Review*, 50, 307-322.

Ausubel, L. (1997): "An Efficient Ascending-Bid Auction for Multiple Objects," mimeo, University of Maryland.

Ausubel, L. (2002): "Implications of Auction Theory for New Issues Markets," Brookings-Wharton Papers on Financial Services.

Ausubel, L., and P. Cramton (1996): "Demand Reduction and Inefficiency in Multi-Unit Auctions," mimeo, University of Maryland.

Ausubel, L., and P. Cramton (1998): "Auctioning Securities," University of Maryland Working Paper.

Back, K., and J. Zender (1993): "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment," *Review of Financial Studies*, 6, 733-764.

Bajari, P., and A. Hortascu (2003): "The winner's curse, reserve prices, and endogenous entry: empirical insights from eBay auctions," *RAND Journal of Economics*, 34, 329-355.

Bajari, P., and G. Summers (2002): "Detecting Collusion in Procurement Auctions," *Antitrust Law Journal*, 70, 143-170.

Balakrishnan, N., and S. Gupta (1998): "Higher Order Moments of Order Statistics from Exponential and Right-truncated Exponential Distributions and Applications to Life-testing Problems," in: Balakrishnan, N., and C. Rao (eds.), *Handbook of Statistics 17 (Order Statistics: Applications)*. North-Holland: Elsevier.

Balakrishnan, N., and C. Rao (1998): "Order Statistics: An Introduction," in: Rao, C. (ed.), *Handbook of Statistics 16 (Theory & Methods)*. North-Holland: Elsevier.

Berney, R. (1964): "The Auction of Long-Term Government Securities," *Journal of Finance*, 19, 470-482.

Biais, B., and A. Faugeron-Crouzet (2002): "IPO Auctions: English, Dutch, ... French, and Internet," *Journal of Financial Intermediation*, 11, 9-36.

Black, J., and D. de Meza (1992): "Systematic Price Divergences Between Successive Auctions Are No Anomaly," *Journal of Economics and Management Strategy*, 1, 607-628.

Bose, S., and A. Daripa (2006): "Optimal Sale: Auction with a Buy-Now Option," Birbeck Working Papers in Economics & Finance.

Brusco, S., and G. Lopomo (2002): "Collusion via Signaling in Simultaneous Ascending Bid Auctions with Heterogeneous Objects, With and Without Complementarities," *Review of Economic Studies*, 69, 407-436.

Budish, E., and L. Takeyama (2001): "Buy prices in online auctions: irrationality on the internet?" *Economic Letters*, 72, 325-333.

Bulow, J., and P. Klemperer (2002): "Prices and the Winner's Curse," *RAND Journal of Economics*, 33, 1-21.

Capen, E., R. Clapp, and W. Campbell (1971): "Competitive Bidding in High-Risk Situations," *Journal of Petroleum Technology*, 23, 641-653.

Cassady, R. (1967): *Auctions and Auctioneering*, Berkeley, CA: California University Press.

Chakraborty, I., and G. Kosmopoulou (2004): "Auctions with shill bidding," *Economic Theory*, 24, 271-287.

Chan, T., V. Kadiyali, and Y. Park (2006): "The Exercise of Buy-It-Now Pricing in Auctions: Seller Revenue Implications," Washington University Working Paper.

Chen, J., K. Chen, and C. Chou (2006): "A Dynamic Model of Auctions with Buy-Out: Theory and Evidences," National Taiwan University Working Paper.

Cramton, P., and J. Schwartz (2000): "Collusive Bidding: Lessons from the FCC Spectrum Auctions," *Journal of Regulatory Economics*, 17, 229-252.

Dasgupta, P., and E. Maskin (2000): "Efficient Auctions," *Quarterly Journal of Economics*, 115, 341-388.

Durham, Y., M. Roelofs, and S. Standifird (2004a): "eBay's Buy-It-Now Function: Who, When, and How," *Topics in Economic Analysis & Policy*, 4.

Durham, Y., M. Roelofs, and S. Standifird (2004b): "The Impact of eBay's Buy-It-Now Function on Bidder Behavior," *International Journal of Electronic Commerce*, 9, 167-176.

Engelberg, J., and J. Williams (2005): "A License to Shill: Incremental Bidding in eBay's Proxy System," Kellogg School of Management Working Paper.

Engelbrecht-Wiggans, R. (1980): "Auctions and Bidding Models: A Survey," *Management Science*, 26, 119-142.

Engelbrecht-Wiggans, R., and C. Kahn (1998a): "Multi-unit Auctions with Uniform Prices," *Economic Theory*, 12, 227-258.

Engelbrecht-Wiggans, R., and C. Kahn (1998b): "Multi-Unit Pay-Your-Bid Auctions with Variable Awards," *Games and Economic Behavior*, 23, 25-42.

Gilkeson, J., and K. Reynolds (2003): "Determinants of internet auction success and closing price: An exploratory study," *Psychology and Marketing*, 20, 537-566.

Graham, D., and R. Marshall (1987): "Collusive Bidder Behavior at Single-Object Second-Price and English Auctions," *Journal of Political Economy*, 106, 1302-1319.

Grebe, T., R. Ivanova-Stenzel, and S. Kröger (2006): "How eBay Sellers set "Buy-it-now" prices - Bringing The Field Into the Lab," Humboldt University Discussion Paper.

Gupta, S., and J. Gallien (2007): "Temporary and Permanent Buyout Prices in Online Auctions," MIT Working Paper.

Harstad, R., and M. Rotkopf (1995): "Withdrawable Bids as Winner's Curse Insurance," *Operations Research*, 43, 983-994.

Hausch, D. (1986): "Multi-Object Auctions: Sequential vs. Simultaneous Sales," *Management Science*, 32, 1599-1610.

Hayne, S., C. Smith, and L. Vijayasarathy (2003): "Who Wins on eBay: An Analysis of Bidders and Their Bid Behaviours," *Electronic Markets*, 13, 282-293.

Hendricks, K., I. Onur, and T. Wiseman (2005): "Preemption and delay in eBay auctions," University of Texas Working Paper.

Hidvégy, Z., W. Wang, and A. Whinston (2006): "Buy-price English auction," *Journal of Economic Theory*, 129, 31-56.

Jehiel, P., and B. Moldovanu (2001): "Efficient Design with Interdependent Valuations," *Econometrica*, 69, 1237-1259.

Jung, H., and B. Kim (2004): "The Role of the Buyout Price in Internet Auctions," *Seoul Journal of Business*, 10, 17-26.

Kagel, J., and D. Levin (1986): "The Winner's Curse and Public Information in Common Value Auctions," *The American Economic Review*, 76, 894-920.

Kandel, S., O. Sarig, and A. Wohl (1999): "The demand for stocks: an analysis of IPO auctions," *Review of Financial Studies*, 12, 227-247.

Kauffman, R., and C. Wood (2005): "The effects of shilling on final bid prices in online auctions," *Electronic Commerce Research and Applications*, 4, 21-34.

Kirkegaard, R., and P. Overgaard (2007): "Buy-Out Prices in Auctions: Seller Competition and Multi-Unit Demands," University of Aarhus Working Paper.

Klemperer, P. (1999): "Auction Theory: A Guide to the Literature," *Journal of Economic Surveys*, 13, 227-286.

Klemperer, P. (2004): *Auctions: Theory and Practice*, Princeton, New Jersey: Princeton University Press.

Klumpp, T., and M. Ranger (2006): "Bid Revisions in the Buy-Price English Auction," University of Indiana Working Paper.

Krishna, V. (2002): *Auction Theory*, San Diego, California: Elsevier.

Lee, H., and B. Ahn (2004): "Buy-Price Options at Online Auctions in the Presence of Competing Fixed-Price Markets," Korea Advanced Institute of Technology Working Paper.

Lopomo, G. (1998): "The English Auction is Optimal among Simple Sequential Auctions," *Journal of Economic Theory*, 82, 144-166.

Lucking-Reiley, D. (2000): "Auctions on the internet: what's being auctioned and how?" *Journal of Industrial Economics*, 48, 227-252.

Maskin, E. (1992): "Auctions and Privatization," in: Siebert, H. (ed.), *Privatization: Symposium in Honor of Herbert Giersch*. Tübingen: Mohr (Siebeck), 115-136.

Maskin, E. (2001): "Auctions and Efficiency," mimeo, Princeton University.

Maskin, E., and J. Reiley (1984): "Optimal Auctions with Risk Averse Buyers," *Econometrica*, 52, 1473-1518.

Mathews, T. (2004): "The Impact of Discounting on an Auction with a Buyout Option: a Theoretical Analysis Motivated by eBay's Buy-It-Now Feature," *Journal of Economics*, 81, 25-52.

Mathews, T. (2006): "Bidder Welfare in an Auction with a Buyout Option," California State University Working Paper.

Mathews, T., and B. Katzman (2006): "The role of varying risk attitudes in an auction with a buyout option," *Economic Theory,* 27, 597-613.

McAfee, P., and J. McMillan (1987): "Auctions and Bidding," *Journal of Economic Literature*, 25, 699-738.

McAfee, R., and J. McMillan (1992): "Bidding Rings," *American Economic Review*, 82, 579-599.

Menezes, F., and P. Monteiro (2005): *An Introduction to Auction Theory,* Oxford, UK: Oxford University Press.

Milgrom, P. (1987a): "Auction Theory," in: Bewley, T.F. (ed.), *Advances in Economic Theory: Fifth World Congress*. Cambridge, UK: Cambridge University Press.

Milgrom, P. (1987b): "Auctions and Bidding: A Primer," *Journal of Economic Perspectives*, 3, 3-22.

Milgrom, P. (2006): *Putting Auction Theory to Work*, Cambridge, UK: Cambridge University Press.

Milgrom, P., and R. Weber (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, 1089-1122.

Milgrom, P., and R. Weber (2000): "A Theory of Auctions and Competitive Bidding II", in: Klemperer, P. (ed.), *The Economic Theory of Auctions, Vol. II.* Cheltenham, UK: Edward Elgar, 179-194.

Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58-73.

Noussair, C. (1995): "Equilibria in a Multi-Object Uniform Sealed Bid Auction with Multi-Unit Demands." *Economic Theory*, 5, 337-351.

Ockenfels, A., D. Reiley, and A. Sadrieh (2006): "Online Auctions", in: Hendershott, T. (ed.), *Handbook of Economics and Information Systems*, Berkeley, CA: Elsevier.

Ockenfels, A., and A. Roth (2006): "Late and multiple bidding in second price Internet auctions: Theory and evidence concerning different rules for ending an auction," *Games and Economic Behavior*, 55, 297-320.

Ortega Reichert, A. (1968): "Models for Competitive Bidding Under Uncertainty," PhD thesis, Stanford University (and Technical Report No. 8, Department of Operations Research, Stanford University).

Peeters, R., M. Strobel, D. Vermeulen, and M. Walzl (2007): " 'Buy it Now or Never!' An experimental investigation into buy-options in online auctions," Universiteit Maastricht Working Paper.

Pesendorfer, W., and J. Swinkels (2000): "Efficiency and Information Aggregation in Auctions," *American Economic Review*, 90, 499-525.

Peters, M., and S. Severinov (2006): "Internet auctions with many traders," *Journal of Economic Theory*, 130, 220-245.

Rasmusen, E. (2006): "Strategic Implications of Uncertainty over One's Own Private Valuation in Auctions," *Advances in Theoretical Economics*, 6.

Reiley, J., and W. Samuelson (1981): "Optimal Auctions," *American Economic Review*, 71, 381-392.

Reynolds, S., and J. Wooders (2005): "Auctions with a Buy Price," University of Arizona Working Paper.

Robinson, M. (1985): "Collusion and the Choice of Auction," *RAND Journal of Economics*, 16, 141-145.

Shariar, Q., and J. Wooders (2006): "An Experimental Study of Auctions with a Buy Price Under Private and Common Values," University of Arizona Working Paper.

Sherman, A. (2005): "Global trends in IPO methods: Book building versus auctions with endogenous entry," *Journal of Financial Economics*, 78, 615-649.

Sinha, A., and E. Greenleaf (2000): "The Impact of Discrete Bidding and Bidder Aggressiveness on Sellers' Strategies in Open English Auctions: Reserves and Covert Shilling," *Market Science*, 19, 244-265.

Smith, V. (1966): "Bidding Theory and the Treasury Bill Auction: Does Price Discrimination Increase Bill Prices?" *Review of Economics and Statistics*, 48, 141-146.

Song, J., and J. Baker (2007): "An integrated model exploring sellers' strategies in eBay auctions," *Electronic Commerce Research*, 7, 165-187.

Stryszowska, M. (2005): "Last-minute and Multiple Bidding in Simultaneous and Overlapping Second Price Internet Auctions," Tilburg University Working Paper.

Thaler, R. (1988): "Anomalies: The Winner's Curse," *Journal of Economic Perspectives*, 2, 191-202.

Thiel, S. (1988): "Some Evidence on the Winner's Curse," *American Economic Review*, 78, 884-895.

Vickrey, W. (1961): "Counterspeculation, Auctions and Competitive Sealed Tenders", *Journal of Finance*, 16, 8-37.

Vickrey, W. (1962): "Auction and Bidding Games," in: *Recent Advances in Game Theory*, Princeton Conference Series, 29, Princeton, New Jersey: The Princeton University Press, 15-27.

Wang, R. (1993): "Auctions versus Posted-Price Selling," *American Economic Review*, 83, 838-851.

Wang, J., and J. Zender (1993): "Auctioning Divisible Goods," *Economic Theory*, 19, 673-705.

Wang, X., A. Montgomery, and K. Srinivasan (2004): " When Auctions Meets Fixed Price: A Theoretical and Empirical Examination of Buy-it-Now Auctions," Carnegie Mellon University Working Paper.

Weber, R. (1983): "Multi-Object Auctions," in: Engelbrecht-Wiggans, R., M. Shubik, and R. Stark (eds.), *Auctions, Bidding, and Contracting*, New York: New York University Press, 165-191.

Wilson, R. (1969): "Competitive Bidding with Disparate Information," *Management Science*, 15, 446-448.

Wilson, R. (1977): "A Bidding Model of Perfect Competition," *Review of Economic Studies*, 44, 511-518.

Wilson, R. (1979): "Auctions of Shares," *Quarterly Journal of Economics*, 93, 675-689.

Wilson, R. (1992): "Strategic Analysis of Auctions," in: Aumann, R., and S. Hart (eds.), *Handbook of Game Theory, Volume 1*, Amsterdam: Elsevier, 228-279.

Wolfstetter, E. (1996): "Auctions: An Introduction," *Journal of Economic Surveys*, 10, 367-420.

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